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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

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Electronic Guidebook of Methods and Tools for teacher facilitators

PART B

A collection of Good Practices and Methods used to support gifted/talented pupils in schools

References link:

https://drive.google.com/drive/folders/1LQAkjsh9F-n9iVa7V7FqglHOFDM3C7rh?usp=sharing

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PART B Introduction

This part of the Guidelines gathers good practices related to the education, development and stimulation of mathematically talented students by providing a collection of mathematics problem for supporting gifted/talented students separated into two age groups, under 14 and over 14 years old. The collection includes problems and applications, competitions and communication activities and related videos. To this end, a structure was designed to facilitate the presentation of the above material by providing a short abstract description following by a file reference that can be found with the same name in the link google drive file given in the link here below. For better access the reader is invited to copy the link and paste it into the url to open the folder.
https://drive.google.com/drive/folders/1LQAkjsh9Fn9iVa7V7FqglHOFDM3C7rh?usp=sharing

PART B - Chapter 1: A collection of Mathematics Problems for gifted/talented school students (age under 14)

In this chapter we've gathered examples of problems dedicated to talented students who are under 14. Among them you can find issues from many fields of mathematics, often related to other disciplines or topics encountered in everyday life. We wanted to show problems dedicated to students with special mathematical talent in a wide range of applications. All materials contained in this chapter have been collected and sent by Partners. They were selected on the basis of criteria established and approved during MEETING 1. This collection will select innovative practices and models that the user can decide what best fit its environment and culture and decide what can work and what cannot. This approach is mainly the innovative character that the project. The main reason for choosing that practice was: to encourage to think, to control the level of comprehension, knowledge, abilities, to focus on the task, to remind and repeat, to enable students to express their feelings, involvement, opinions, to provide an opportunity to answer questions, to encourage shy students to take part in the discussion, to teach by using students' answers, to ask gifted students as the way to stimulate others. We tried to make the found materials fit into previously prepared

categories: concern one particular field, concern more than one field, concern one particular ability, concern more than one ability, serve as a revision, serve as an ability practice, bring new knowledge for the student, develop new abilities, building self-confidence. Topics of these problems include:

B1.1 The material concerns work in the field of geometry. Students have to use mainly the Pythagorean Theorem. There are several approaches possible. Students can start working directly at the problem and find solutions by themselves. Or they can use the given answers, of which exactly one is not correct, and deduce which is the wrong one. So it provides an excellent context for reflective thinking and deductive reasoning abilities. The complete solution is given.

B1.1.1 Geometry-Snowman.pdf

B1.2 The material concerns work in the field of stochastics and testing. Students might to use binomial distribution. However, they might as well use intuitive approaches. There are several approaches possible. Students can start working directly at the problem and find solutions by themselves. Or they can use the given answers, of which exactly one is not correct, and deduce which is the wrong one. So it provides an excellent context for reflective thinking and deductive reasoning abilities. The complete solution is given.

B1.2.1 Stochastics-Fairy-light.pdf

B1.3 The material concerns work in the field of geometry and packing problems. Students might to use intuitive sketches to get first ideas and confirm the ideas by the Pythagorean theorem. So it provides an excellent context for straightedge and compass constructions joined by deductive reasoning abilities. The complete solution is given.

B1.3.1 Packing-problem.pdf

B1.4 The problem concerns the work in the field of geometry. A problem-centred approach was used to identify data and mathematical relationships. Completing the learning unit "Triangle Similarity" aims to provide the student with skills that allow him to: a) express ideas about similarity in the vocabulary of proportions b) application of results-fundamental

theorem of similarity, similarity criteria, bisector theorem-in solving of practical problems. Students will acquire the skills necessary to identify pairs of similar triangles in given geometric configurations and will establish similarity relations between two triangles by different methods. The teacher will interpret the similarity of triangles in correlation with qualitative and / or metric properties and will apply the similarity of triangles and the bisector theorem in solving mathematical or practical problems.

B1.5 The problem is in the field of geometry. A problem-centred approach was used to know and understand the concepts, terminology and calculation procedures specific to the centre of gravity of the triangle and the medians. The purpose of completing the learning unit is to develop the students' ability to exploit / investigate the properties of the medians and the centre of gravity in a triangle. The teacher will facilitate the development of the students' work capacity, using the mathematical language specific to the problems in which the cases of congruence of triangles are used. It is essential to develop interest and motivation for the study and application of geometry in various contexts, especially for talented students and especially interested in geometry. Students can pose challenging problems and work independently or in groups.

B1.5.1 The centre of gravity of a triangle.pdf

B1.6 The problem is algebra, the divisibility chapter. In solving this problem, students are instructed to use the knowledge gained in the study of calculus with powers of natural numbers and the use of the last digit of the power of a natural number. The teacher points out that the last digit of the powers of 2 and 3 is repeated 4 by 4 and therefore the natural numbers m and n are considered in a certain form. By solving problems of this type, students will acquire the skills necessary to identify the criteria of divisibility in the multitude of natural numbers. It is essential to be aware at the level of students talented in mathematics, the fact that understanding the concepts, terminology and calculation procedures specific to the criteria of divisibility is important in solving such problems. Students can ask challenging questions for further study and can work in groups that include more students interested in the arithmetic of natural numbers.

B1.6.1 Divisibility.pdf

B1.7 The MATHEU Volume 2 is a set of mathematics problems for gifted students under the age of 14 and they are designed in a latter form of difficulty. The problems are the outputs of the EU funded project MATHEU. The design is made so the higher the level reached the more talented the student is. At the same time the climbing of the latter serves as a motivation to students to reach higher level. In this manual the reader can find problem covering the following Mathematics topics: Dirichlet Principle, Mathematical Games, Geometry, Inequalities, Invariants, Number Theory, Polynomials and Equations, Arithmetic, Geometric and Harmonic Sequences.

B1.7.1 MATHEU-Volume2.pdf

B1.8 Problem 1 deals with the notion of symmetry, the properties of straight lines and menstruation in polygons. Working through this problem students, have to use their quantitative reasoning and apply their understanding of the notions of space, the connections between them and use geometric properties in real world objects.

B1.8.1 Problem 1 with solution.pdf.

B1.9 Problem 2 deals with the fields of geometry and trigonometry. This problem is a good example to be used for students to develop heuristic strategies in approaching the solution of a problem as there are many methods to work through the solution. Students will need to be able to extract important information from the problem, apply their mathematical reasoning and develop conceptual understanding, recalling and applying geometrical knowledge accurately to the properties of shapes. By being exposed to such problems, students develop their confidence, persistence and organization in problem solving.

B1.9.1 Problem 2 with solution.pdf

B1.10 Problem 3 deals with the properties of numbers and in particular prime numbers. This is an excellent example to train students to be able to formulate their mathematical thinking in mathematical sentences. Students will need to use the properties of operations to express numbers in different forms to assist in solving a problem. Students should be able to reflect on their solution and use their critical thinking to evaluate how rational their answer is based on the given facts.

B1.10.1 Problem 3 with solution.pdf.

B1.11 The material contains a problem of logical conclusions. Problem-solving-abilities are developed by processing given statements, so the students have to find suitable approaches (e.g. tables, hypotheses ...). It provides an excellent context for developing intuition, logical thinking and deductive reasoning abilities.

B1.11.1 Einstein's Riddle.pdf

B1.12 The material concerns problems in the field of geometry and functional relation. A problem-centred approach will help to develop problem-solving abilities, structured thinking, visual thinking and so on. Students can find challenging problems and work independently. The tasks provide insights into real problems that can be solved with mathematical tools.

B1.12.1 Challenging Problems.pdf

B1.13 The materials are a complete proposal for teaching mathematics with a graphic calculator and computer for students aged 12-14. It takes into account the learning objectives, teacher and school tasks and the content of mathematics teaching. Presented Mathematics, Calculators and Computer program, whose general assumption is that during the process of teaching mathematics, i.e. in the process of shaping mathematical concepts, conducting mathematical reasoning, solving tasks and problems and teaching mathematical language, we use computer and calculators at these moments, in which the teacher encounters a variety of problems that are difficult to overcome using traditional teaching methods and teaching aids used so far. In this concept, the computer and calculators are treated as an important didactic means, as an important help in achieving some of the intended goals.

B1.13.1 Matematyka Kalkulatory i Komputery.pdf

B1.14 Materials focus on teaching mathematics to students aged 6-10. This is a complete guide, which shows what are the characteristics of children at a younger school age and how to plan your work, taking into account the development opportunities of children, to comprehensively familiarize them with the world of mathematics. Many of the proposals

presented create situations in which students that are interested and mathematically gifted try to find other non-standard solutions to problems, proving that they follow their own reasoning. Mathematical exercises and tasks collected do not inhibit the development of a student with extraordinary abilities, but allow him to think independently and custom action. They encourage active participation in classes and creative approach to solved tasks.

B1.14.1 Rozwijanie_zainteresowan_i_zdolnosci_matematycznych-6-10.pdf

PART B - Chapter 2: A collection of Mathematics Problems for gifted/talented school students (age over 14)

In this chapter we've gathered examples of problems dedicated to talented students who are over 14. Among them you can find issues from many fields of mathematics, often related to other disciplines or topics encountered in everyday life. We wanted to show problems dedicated to students with special mathematical talent in a wide range of applications. As in the previous chapter, the criteria and rules for searching materials were established during Meeting 1. The questions that drove the search concerned: What kind of practices (methods or materials) is it? Why can teachers and students be interested in these materials? What are the most important reasons for choosing this material? What problems can I solve with these materials? Answers to what questions will be found? What is the main idea in the proposed material? Important factors were: complexity of the answer, credibility of the answer, authenticity of the answer, transparency of the answer, compliance with the requirements, quality of arguments. Topics of these problems include:

B2.1 The material concerns the exciting field of prime numbers. It serves as an excellent overview over the fascinating world of prime numbers for mathematically gifted and talented high school students. Starting with elementary facts about prime numbers, this overview touches at various places also questions that are investigated by current research. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition and reflective thinking.

B2.1.1 primenumbers.pdf

B2.2 The problem is in the field of algebra, the chapter "Equation of the second degree". Students, in solving this problem, use and apply Viete's relations and the condition that the second degree equation admits real solutions, integers. Students use appropriate notations to facilitate calculations and abbreviated calculation formulas. The teacher explains the use of the algorithm and mathematical concepts in finding the solutions of the second degree equation. Mathematical talented students can work independently or in groups, analyzing the methods used. The teacher will find other examples with complete solutions and didactic comments. The problem provides an excellent context for the development of intuitive thinking and deductive reasoning skills.

B2.2.1 Equation of the second degree.pdf

B2.3 The problem is from the chapter "Strings of real numbers", and solving it requires special mathematical skills and the correct use of mathematical algorithms. Students talented in mathematics, will use the notions learned in the chapter "Geometric progressions" and "Inequalities". Guided by the teacher, students will prove that the set is empty, and being made up of natural numbers, admits a minimum. The teacher instructs the students that in order to prove point b) of the problem, they must apply mathematical induction and finally properties of inequalities. The teacher explains the principle of mathematical induction to demonstrate statements of type P (n), any natural n. Mathematical students can work in groups, analyzing and determining the hypotheses needed to obtain the conclusion.

B2.3.1 Strings of real numbers.pdf

B2.4 The problem is in the field of studying derivable functions and their properties. In solving this problem, students will make convenient notations, observing the injectivity of the function g, which is also continuous, so strictly monotonous. The teacher instructs the students how to use the theory of the monotony of derivable functions and the determination of extreme points, using Fermat's theorem. Solving this problem develops in talented students in mathematics, the ability to make connections that allow the most complete explanations regarding the properties of derivable functions. The problem provides useful concepts for math circles that include students interested in math analysis.

B2.4.1 Derivable functions.pdf

B2.5 The proposed problem is from algebra, the chapter "Exponential equations". Students notice that the solutions of the equation are 0 and 1. But then prove the uniqueness using the fact that the graph of a strictly convex function is intersected by a line at most two points. The teacher can propose to the talented students in mathematics an alternative solution invoking the inequality of Bernoulli and the monotony of the function. For the second part of the problem, the students use the injectivity of the function f. Solving the problem aims at developing the capacity to make connections that allow complete explanations regarding the study of functions. Mathematical talented students will be motivated to study and apply mathematics in a variety of contexts.

B2.5.1 Exponential equations.pdf

B2.6 Problems is mathematical analysis, in the chapter "Strings of real numbers". In solving this problem students will use the knowledge regarding geometric progressions, inequalities and will demonstrate the inequality obtained, through the method of mathematical induction. The teacher indicates to the students how to use the theoretical results in the case of the series, selecting appropriate work strategies in solving such problems. Students are provided with an excellent context for developing intuition and for the correct use of algorithms in the case of strings. Mathematical talented students can solve the problem individually, having the opportunity to compare and identify similarities and differences that occur in problems with real number strings.

B2.6.1 Strings of real numbers2.pdf

B2.7 The problem is vector geometry. In solving this problem, students are put in the situation to apply the theoretical knowledge regarding vectors, operations with vectors, non-linear vectors. The teacher guides the talented students in mathematics to determine the center of gravity of the triangle ABC, specifying its vector property. Together with the students, the teacher determines the hypotheses needed to obtain the conclusion. students who can also make generalizations.

B2.7.1 Vectors.pdf

B2.8 The problem is in the field of algebra, the chapter "Matrices and determinants". The advantages of solving such problems are multiple: a higher level of thinking is formed; a strong motivation appears; interpersonal and group understanding is promoted. Students, in solving the problem, apply properties and calculation rules of the determinants, write correctly in formal language the solution or strategies for solving such a problem. The teacher explains the use of algorithms and mathematical concepts in solving problems with matrices, matrix traces and determinants. Students can find problems challenging and they can work independently or in groups. It provides an excellent context for the development of intuitive thinking and deductive reasoning skills. The problem provides useful concepts for mathematics circles that include students interested in algebra.

B2.8.1 Matrix and determinants.pdf

B2.9 The problem is in the field of the set of complex numbers, properties and operations with complex numbers. Students are familiar with the notion of complex number, which is not the result of a measurement, as in the case of a real number, but although it has an abstract character, has multiple practical applications in fields such as electrical engineering, telecommunications, mechanics, astronomy, atomic physics, etc. are guided to choose solving strategies in order to optimize calculations with complex numbers. The teacher guides talented students in mathematics to determine analogies between the properties of operations with complex numbers written in various forms and their use in solving problems.

B2.9.1 Complex Numbers.pdf

B2.10 The problem concerns the activity in the field of studying the functions and their properties. In solving this problem, students are put in a position to apply the definition of even and odd functions. The purpose is to form the habit of resorting to different types of representations of the data contained in mathematical statements as a starting point for intuition and justification of ideas or methods of solving. The teacher indicates to the students how to use the results and the solving methods in order to select appropriate work strategies in solving problems with different types of functions. The aim is to develop interest and motivation for the study and application of mathematics in various contexts, especially for students talented in mathematics.

B2.10.1 Functions seem; odd functions.pdf

B2.11 The MATHEU Volume 2 is a set of mathematics problems for gifted students over the age of 14 and they are designed in a latter form of difficulty. The problems are the outputs of the EU funded project MATHEU. The design is made so the higher the level reached the more talented the student is. At the same time the climbing of the latter serves as a motivation to students to reach higher level. In this manual the reader can find problem covering the following Mathematics topics: Dirichlet Principle, Mathematical Games, Geometry in the Plane, Mathematical Induction, Inequalities, Linear Algebra, Number Theory, Transformation Methods, Complex number in Geometry, Sequences.

B2.11.1 MATHEU-Volume3.pdf

B2.12 This problem falls in the intersection of logical thinking and computational mathematics. This is a good example in helping students develop problem solving skills, apply structured and logical thinking as well as deductive reasoning. Personal attributes such as confidence, persistence and organization in approaching a problem are also developed. This problem is a great chance for students to apply their prior knowledge in several notions of mathematics in order to reach a solution.

B2.12.1 Problem 1 with solution.pdf

B2.13 This problem deals with inductive and deductive reasoning and critical thinking. It involves analytical thinking and expects the students to construct logical arguments, expose illogical arguments and re-evaluate their resulting argument. It promotes rational and logical thinking. This problem is also a good opportunity for students to be asked to work not only individually but in groups as well in order to improve their skills in communicating a mathematical solution as well as commenting and reviewing on the reasoning of their peers.

B2.13.1 Problem 2 with solution.pdf

B2.14 This problem deals with mathematical reasoning, analytical thinking and optimization. It promotes the students' ability to combine pieces of information to form general rules or conclusions. This problem is good for differentiating the students' ability to

choose the right mathematical method to approach the problem, as some approaches might be time consuming. This problem is ideal in promoting heuristic strategies in problem solving. It also promotes the skills of communicating a solution which is often a difficulty students are met with; how to explain their conclusion with a mathematical idea.

B2.14.1 Problem 3 with solution.pdf

B2.15 The material concerns the exciting field of prime numbers. It serves as an excellent overview over the fascinating world of prime numbers for mathematically gifted and talented high school students. Starting with elementary facts about prime numbers, this overview touches at various places also questions that are investigated by current research. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition and reflective thinking.

B2.15.1 Prime-numbers.pdf

B2.16 The material deals with a problem of computing optimal timetables in public transit networks. It provides a good opportunity to get insights in the field of mathematical optimization. The students will develop or enrich their skills of logical conclusions. Furthermore, they can train their problem-solving abilities by processing given statements of conditions of a fictive timetable.

B2.16.1 Rudolph's timetable.pdf

B2.17 The material contains a problem of an optimal auction for multiple goods. It provides insights in the field of tropical geometry, an application of the probability theory. The students will enrich their skills of the computing of probabilities, prices and expected revenues. In addition, the teachers can use it as an application-oriented example in the field of business mathematics.

B2.17.1 Everything must go!.pdf

B2.18 The material concerns the field of cryptography. It serves as an excellent introduction to the field of asymmetric cryptosystems and addresses high school students with interest in number theory; in particular, a detailed description of the RSA public-

key cryptosystem is given. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition, reflective thinking, and deductive reasoning abilities. The teaching and learning material is particularly useful for mathematically gifted and talented high school students.

B2.18.1 RSA encryption.pdf

B2.19 The materials present teachers with a number of proven possibilities for effective learning of mathematics, instead of teaching how to play algorithms for standard exam tasks. All proposals are checked in practice. It is certainly worth trusting this guide and try to instill the ideas presented in it on your own foundations, adapting them to your needs. Mathematical interests and talents of students are rarely communicated to teachers. In most cases they must earn it solidly thanks to enthusiasm, commitment and ingenuity and professionalism. This guide will be helpful because it shows you how to work with ordinary students, arousing their interest and motivation to become extraordinary students. The authors try to answer questions on how to educate thinking, creative and inventive students, how to instil mathematical passions in them and how to develop interests, and finally - how to educate key mathematical skills: logical thinking, precise argumentation and use algebraic techniques and perception of geometric relationships.

B2.19.1.jak_pracowac_z_uczniem_zdolnym_poradnik_nauczyciela_matematyk.pdf

B2.20 The materials present the concept of teaching the probability theory based on the concept of a stochastic graph. Probability theory today has more and more applications in various areas of human life. The textbook gives a different perspective on this section of mathematics. Basic probabilistic concepts are introduced in the context of generalizations of Penney-Ante games. The basic tool in solving various problems is the stochastic graph. Thanks to this, the student gains an interesting, intuitive and very effective conceptual apparatus.

B2.20.1 Tlusty, Krech – MONOGRAFIA.pdf.

PART B - Chapter 3: Projects and Applications

This section includes older completed projects of different types and current running projects. Like the previous two sections, this one was created as a result of cooperation of Partners in the scope of searching for projects and applications helpful in working with a talented/gifted students. The found materials are to serve as a models for implementation and also to show recent project results and case studies of practitioners. The spectrum of interest includes such materials as: data base with tasks and exercises (without solutions), data base with tasks and exercises (with solutions), tutorial given by a teacher, tutorial given by a student, forum moderated by a specialist, forum moderated by a student, interactive media, interactive media, individual games, cooperative or collaborative games, project problems, competitions. To make the review of materials easier and more useful for the reader, we have divided it into subsections: Projects/Investigation and Applications related to other sciences(STEAME) and real life, Projects/Investigation and applications related to gaming, projects/Investigation and applications related to industry/business world/thematic/research institutes/authorities/organizations relating to real life issues, Projects and applications related to cooperation with universities.

B3.1 Projects/Investigation and Applications related to other sciences(STEAME) and real life

B3.1.1 GOLDEN RATIO - NUMBER AND UNIVERSE is a complex and elaborate project, which involved students aged between 16 and 19 from Italy and Romania. The project aimed to arouse students' interest and curiosity in mathematics, proving that it has always been an essential part of human development and culture. The students were tasked with searching for information about the gold ratio and the Fibonacci series; notation of links and definitions of new notions; group activities in order to clarify the theoretical concepts; developing applications in collaboration with teammates; evaluating and, possibly, correcting some applications of the other colleagues; participating in discussions with coordinating teachers.

B3.1.1.1 A GOLDEN RATIO - NUMBER AND UNIVERSE - EN.pdf

B3.1.2 The ratio of the Gifted European Mathematicians (G.E.M.) project was born from the recognition of the need for many European countries for highly qualified youngers in mathematics, science and technology, that are available to mobility, with high language skills and able to propose innovative solutions for the social and economic growth. The context of the school partners in G.E.M. arises from the needs identified on the target group (students 14-17 years old) and identifies two topics with similar needs: Maths Literacy, English scientific skills.

B3.1.2.1 Gifted European Mathematicians (G.E.M.).pdf

B3.1.3 The “Stiftung Planetarium Berlin” unites the astronomical institutions of Berlin. The Archenhold-Sternwarte and the Wilhelm-Foerster-Sternwarte are among the most traditional public observatories in Germany, while the Planetarium am Insulaner and the Zeiss-Großplanetarium, the most modern science theater in Europe, offer relevant and innovative forms of education. As a joint institution, they are an essential part of Berlin’s educational and cultural offerings. In addition to astronomy, the range of topics will be extended to other sciences. In the near future, materials for mathematics workshops accompanying the astronomical events of the planetariums shall be offered in cooperation with the Humboldt-Universität zu Berlin.

B3.1.3.1 Planetarium.pdf

B3.1.4. Zero Robotics is a space-based programming competition for middle and high school students. Massachusetts Institute of Technology (MIT) runs the program in collaboration with NASA and CASIS. Each year features a new “game” based on a real-life space exploration challenge selected by MIT and NASA. Competing student teams must program their robots to manoeuver with precision and strategy, protecting their own robot while trying to thwart their opponents’ efforts. The competition uses soccer-ball-sized robots, that float inside the International Space Station and manoeuver with small puffs of compressed gas.

B3.1.4.1 Zero Robotics.pdf

B3.1.5 F1 in Schools is the only global multi-disciplinary challenge in which teams of students aged 9 to 19 deploy CAD/CAM software to collaborate, design, analyse, manufacture, test, and then race miniature compressed air powered F1 cars. Providing opportunities to learn Science, Technology, Engineering and Maths (STEM) related subjects in an exciting way. The challenge inspires students to use IT to learn about physics, aerodynamics, design, manufacture, branding, graphics, sponsorship, marketing, leadership/teamwork, media skills and financial strategy, and apply them in a practical, imaginative, competitive and exciting way. During the development of their cars teams need to follow a handbook of regulations followed in the real F1.

B3.1.5.1 f1inschools.pdf

B3.1.6 NASA Space App Challenge. This Challenge takes place in Nicosia Cyprus every year. Participant teams had to prepare and present a project entitle: How would you do about colonising the moon, which included defining the contributing members of the base, designing a list of requirements for countries to contribute in such a project, decide how many people from each country or state should participate, structure a leadership and governance for the colony(ies), design a social structure etc. The teams had one weekend to think about their project and ten minutes to present it before the judges, two from the CSEO and one from NASA. NASA offers the winning team the unique opportunity to visit the Marshall Space Flight Centre where astronauts are trained, while three more teams will visit the Rocket and Space Centre. All teams will be given the chance to train like astronauts for one week. Every year students are given a new topic to work on.

B3.1.6.1 Spaceappschallenge.pdf

B3.2 Projects/Investigation and applications related to gaming

B3.2.1 Math in Wonderl@nd: with only two partner schools, from Italy and Romania, this European Mathematics and Science Prize-winning project started with the mathematical allusions in "Alice in Wonderland" to show students that math can be " friendly", full of mystery and imagination, to help them discover and develop their ICT, social, linguistic skills and creativity through collaborative activities and active learning. Within the project mathematical games were developed Because Mathematics is such a serious subject, it has become a pleasant and friendly discipline by creating games developed in the Mathematical Games section of the online magazine, as well as on the blog. The project aims to enhance the students' motivation by making Mathematics a learner-friendly subject, to help them discover and develop their creative, social, language and ICT skills, by means of collaborative work and active learning.

B3.2.1.1 Math in Wonderl@nd CCDTR_EN.pdf

B3.2.2 Mathe im Advent – Making Math fun and relevant. The German Mathematical Society (DMV) runs this online Advent Calendar each year in Christmas time for two age groups: From grades 4 to 6 and 7 to 9, respectively. With interesting mathematical problems and concepts, which are usually not taught at schools, it reveals to more than 100,000 school kids the often-surprising message: Math can be fun and useful!

B3.2.2.1 Mathe-im-Advent.pdf

B3.2.3 The MATH+ advent calendar offers fascinating insights into recent application-oriented mathematical research and into the everyday professional life of mathematicians. The 24 challenging puzzles are aimed at bright students from grade 10-13, university students, teachers and everyone interested. Therefore, the calendar is intended to challenge and support (not only) gifted pupils in mathematics. All participants can discover the power and beauty of mathematics and the best ones can win prizes.

B3.2.3.1 MATH+ advent calendar.pdf

B3.2.4 Math Game Day – Students in teams according to their year group, compete online in real time with students from other schools all over Cyprus and abroad. They are given a series of short calculations (+ - * /) to do in a specific amount of time, each game consists of 3 rounds for each team/year group and it lasts about 20 minutes with breaks.

B3.2.4.1 thalescyprus.pdf

B3.2.5 The idea of a mathematics Hackathon organized either internally or internationally among schools is also a good opportunity for gifted students in mathematics to further expand their mathematical skills but also to be given the opportunity to work as a team.

B3.2.5.1 maths-hackathon.pdf

B3.3 Projects/Investigation and applications related to industry/business world/thematic/research institutes/authorities/organizations relating to real life issues

B3.3.1 The European Light Infrastructure - Nuclear Physics (ELI-NP) project is a project that aims to develop the most powerful laser in the world and create a pan-European research centr in Romania, at Magurele. This is the first of its kind in Europe, the results of the developed technology can have applications in fields such as medicine, nuclear physics, electronics, information technology, pharmaceuticals, waste recycling technology, etc. Extreme Light Infrastructures (ELI) project aims to establish European laser research facilities in Romania, Hungary and Czech Republic for the investigation of high-intensity light-matter interactions and their potential applications in science, industry and medicine. Thales developed with ELI-NP (Nuclear Physics) in Romania an unique laser system made of two beam lines each delivering, a peak power of 10 PW at 1 shot/min, with intermediate outputs at 1 PW, 1 Hz and 100 TW,10 Hz.

B3.3.1.1 European Light Infrastructure project_EN.pdf

B3.3.2 Jugend forscht encourages and supports talented achievers in the areas of science, technology, engineering, and mathematics (STEM). It seeks to inspire young people to become involved for the long term, and, after the contest, to help them with their careers.

“Jugend forscht” is the largest public-private partnership of its kind in Germany. There are about 250 partners, most of them from the business community. They organize the contests, donate the prizes and lend their support to other activities. The Federal Ministry for Education and Research (BMBF) defrays the running expenses of the Jugend forscht head office.

B3.3.2.1 Jugend-forscht.pdf

B3.3.3 Research and business expeditions are expeditions for a team of 15 - 40 German-speaking students to interesting regions (e.g. Iceland, North Cape, Azores ...) in order to experience the research process for themselves. They explore fauna, flora, waters, climate, human-culture-language, infrastructure and economy. This means that all parts of the expedition are experienced during the course of the expedition, from the setting of objectives to the planning of experiments and from data collection to the complete documentation. The young researchers have to carry out experiments, surveys and data collection, arrange, evaluate, and document them and finally present their results.

B3.3.3.1 Research and business expeditions.pdf

B3.3.4 London Science Forum. The London International Youth Science Forum (LIYSF) is a two week residential student event held at Imperial College London & The Royal Geographical Society, with lecture demonstrations from leading scientists, visits to world class laboratories and universities combined with cultural interaction, with 500 students aged 16-21 years old from 70 countries.

B3.3.4.1 liysf.pdf

B3.4 Projects and applications related to cooperation with universities

B3.4.1 One was an Approach to Mathematically Gifted by Dr Boštjan Kuzman Through examples, he presented different situations that can arise while working with mathematically gifted students and can be a cause of embarrassment for the teacher as well as the students. Apart from the mathematical knowledge and appropriate motivation, the teacher who works with mathematically gifted must also provide appropriate and socially secure environment. He was determined to solve a couple of mathematical challenges along the way. Another one

was Scratch & Bebras Puzzles by Dr Irena Nančovska Šerbec & Špela Cerar In recent years, computational thinking has become one of the big issues in education worldwide. Various learning activities and competitions were created for development of computational thinking skills. In this spirit, several summer schools were conducted for primary school pupils using ScratchJr and Scratch.They presented some of the ideas that can be used for developing games and stories with gifted kids. Bebras competition is run in Slovenia from 2011. Lots of different tasks and puzzles were created during this time. They presented some ideas how to use the puzzles for learning of fundamental concepts of computer science. Problem questions: 1) What programming activities in Scratch are interesting for gifted students? 2) Are tasks that teachers find difficult also challenging for gifted students? 3) What makes a task interesting for gifted students?

B3.4.1.1 Summer School of Science and Technology from Măgurele_CCDTR_EN.pdf

B3.4.2 Summer School of Science and Technology from Măgurele aims to enhance the scientific expertise of the education and research platform at Magurele and the University of Bucharest making it accessible to high school students regardless of their profile and specialization and teachers who teach STEM subjects (science, technology, engineering, mathematics) in secondary and secondary education. The school proposes a complex complementary program based on the premise that STEM education through its purposes is a vector of impact on increasing competitiveness and social cohesion, in the context of sustainable development of Romania with significant influences on increasing the welfare of both learners and the community in which they live. The trainers and mentors at the summer school are researchers, teachers from pre-university and university education in the fields of STEM, education sciences but also from other specializations, the approaches being of integrated type.

B3.4.2.1 EGIFT Project-European Gifted Education Training.pdf

B3.4.3 The Mathematische Schülergesellschaft “Leonhard Euler” (MSG) is an extra-curricular, free institution for the promotion of mathematically interested and talented secondary school students. It is organized by the Institute of Mathematics of the Humboldt-Universität zu Berlin in cooperation with other Berlin universities. In about 30 weekly courses

(“circles”) mathematically interested and talented pupils from grades 5 to 12 are given exciting insights into various areas of mathematics beyond the school curriculum. The focus is on problem-oriented work, scientific methods, and training for mathematical competitions.

B3.4.3.1 MSG.pdf

B3.4.4 The Summer Schools “Lust auf Mathematik” take place every year and last one week. During this week interested and talented students from grade 11 work in small groups on challenging mathematical problems. They are instructed by professors and scientists of the Humboldt-Universität zu Berlin and other universities of Germany. At the end of the week the students present their results by a talk and document their work with a 10-page report.

B3.4.4.1 Summer-schools.pdf

B3.4.5 The „Berliner Netzwerk mathematisch-naturwissenschaftlich profilierter Schulen“ is a network of five grammar schools, the Humboldt-Universität zu Berlin and the state government of Berlin. It was founded in 2001. This network is promoting students who are talented in mathematics and the natural sciences. This is achieved by an enriched curriculum in mathematics and additional semester hours starting in grade five. Students learn about mathematical structures, argumentation and proofs. They have the opportunity to obtain credit points for the first semester of university education in mathematics. The grammar schools have approximately 1800 students in classes with a focus on mathematics and the natural sciences.

B3.4.5.1 Berliner Netzwerk mathematisch-naturwissenschaftlich profilierter Schulen.pdf

B3.4.6 The University of Cyprus Department of Mathematics and Statistics organises a workshop for high school students who love Mathematics. This year’s workshop was entitled: ‘Why Math’. Students had the opportunity to listen to interesting speeches from experts and university professors on what they can do with Mathematics, what skills they can develop and what careers they can follow with a degree in Mathematics.

B3.4.6.1 WhyMath.pdf

PARTB - Chapter 4: Competitions/Challenges and Communication activities

B4.1 FutureOlympics is a project realized in partnership between Intuitext and the Romanian Society of Mathematical Sciences. Through this project, the Olympics in mathematics training are supported. Together with the professors from the Romanian Mathematical Sciences Society, keeping the tradition of the Mathematical Gazette, Intuitext brought online the performance in mathematics! Every year, the competition on the FutureOlympics website has several stages (students solve problems, send solutions and receive points depending on performance) and is completed with a camp in Câmpulung, where the best students participate for free and experience mathematics otherwise. The competition is part of the Mathematics Education Otherwise Program, a program of the Romanian-American Foundation implemented in partnership with the Romanian Mathematical Sciences Society.

B4.1.1 FutureOlympics.pdf.

B4.2 The Speed Kangaroo Competition is a group competition based on the world famous Kangaroo competition. Students solve Kangaroo tasks in groups of up to six participants as quickly as possible. The fastest group sets the processing time – but be careful, for every wrong answer the other groups get additional time.

B4.2.1 Speed-Kangaroo-Competition.pdf

B4.3 The Berlin Day of Mathematics is a mathematical event that takes place at a different university in Berlin every year since 1995. It is aimed at students and teachers who are interested and gifted in mathematics. The aim of this event is to maintain and develop enthusiasm for this science with exciting tasks, inspiring lectures and interesting workshops about the fascinating world of mathematics. Furthermore, team competitions will be held on this day so that social and mathematical skills are supported by solving challenging tasks.

B4.3.1 Berlin Day of Mathematics.pdf

B4.4 The Mathematics Olympiad in Germany is a competition for (gifted) students in mathematics from grades 3 to 13. In this competition, students have the opportunity to prove and develop their mathematical skills. Challenging tasks promote logical thinking, reasoning abilities, the ability to combine and the creative use of mathematical methods. The competition extends over several rounds - those who get to the federal level can hope to win Olympic gold. The best participants will be delegated as the German team to the International Mathematical Olympiad.

B4.4.1 Mathematics Olympiad in Germany.pdf

B4.5 Regional Mathematics Competition – students have two hours and are expected to work on about 4 questions that require a structured answer. Also, the Mathematics Olympiad organized by the Cyprus Mathematical Society is a good competition for gifted students in Mathematics to compete, show their abilities in Mathematics and gain experiences for future competitions regarding University entries and university life.

B4.5.1 competition-cms#competitions.pdf.

B4.6 Math Relay Competition – Students are in teams according to their year group. We have three teams (Years 7 - 9) each consisting of three students. Then the Year 7s get 15 minutes to answer one question that requires a structured answer. When they finish Year 8s have 30 minutes to answer their own question and Year 9s have 45 minutes for their own. In case a group has a few minutes that they did not use the next team can use this time, and that is why it is a math relay.

B4.6.1 mathematics-Relay.pdf.

B4.7 Math Factor and Science Factor competitions within the framework of Euromath & Euroscience Student Conference: Students need to communicate/present a mathematical or scientific concept in a funny and simple way so that it is comprehensible to even a non - mathematical or scientific oriented audience. Also Math Theatre and Science Theatre Competitions are an excellent mideun for students to communicate science and mathematics.

B4.7.1 euromath.pdf

B4.8 Krakow Youth Society of Friends of Sciences and Arts at the Youth Center dr. H. Jordana and the Cracow Branch of the Polish Mathematical Society invite you to the MAŁOPOLSKA MATHEMATICAL COMPETITION ending with the Małopolska Mathematical Session. We encourage students from secondary and primary schools to participate in the event. Every year, the competition gathers enthusiasts of mathematics from the entire Małopolska province. Some do their work alone, others choose to work in a group. The event provides for two forms of work implementation: individual and team (a team cannot have more than 3 people). Below are sample topics for papers: Presentation of the problem which is dealt with by young people during mathematics club classes. You can present methods (also ineffective) that tried to solve the problem. Presentation (in a way that is understood by peers) of theories or concepts from extracurricular mathematics. Solution to an interesting task. You can present different methods to solve the same task. Unusual, unknown evidence of known mathematical facts Development or implementation of scientific support or measures to popularize mathematics among peers. The awarded works are published on the website www.timediawo.edu.pl, which can help in choosing the topic of work.

B4.8.1 małopolski_konkurs_prac_matematycznych.pdf

PARTB - Chapter 5: Videos and Interactive Media related to giftedness

B5.1 A solution that they propose for this situation is the online mathematics course, based on 40 video-learning units for different topics (ex: Circle, Rectangle, Sequences, Factors and multiples, etc.). This course can be freely accessed by anyone who is interested in mathematics – students, teachers, parents. It can be accessed by any device, from your class or from your home. You can access the online mathematics course from the e-learning platform Menu Cursuri/Mathematics Open Online Course. To access this course you must create a username and password. After these 3 steps, you gain access to 40 video tutorials about maths and more than 200 exercises. Everything is free! The 40 video tutorials, available in both English and Romanian Language, can also be accessed from the YouTube Channel dedicated.

B5.1.1 mathematics-open-online-course.pdf

B5.2 Gifted and talented children: signs and identification. In this video, parents of gifted children talk about noticing early signs like a big vocabulary, advanced use of language or musical talent. They talk about having mixed feelings and working out what it means for their child's future. An educational psychologist says it can help to get a formal assessment when a gifted child is approaching school age, to help the school plan for your child's educational needs.

B5.2.1 Gifted and talented children- signs and identification.pdf

B5.3 Gifted, creative and highly sensitive children. Being "gifted" often feels far from a good thing, when you're the one living it. And yet the myth persists that "gifted" is an elitist and privileged label in education. That "those kids" will be fine without any extra programs or supports. After all, what's the worst that can happen – they get bored? In reality, many of our most vulnerable learners are those "gifted" kids who know they just don't "fit it." In our rush to identify and label children for purposes of funding and organizing our educational services most efficiently, we're missing the ground level, actionable and everyday needs of an entire grouping of needy kids. Our most gifted, sensitive and creative students are our canaries in the coal mine; they are more sensitive to the world around them and the reactions we see in them can teach us what our education system needs to be, believe, include and stop. As is often the case, what is necessary for some is beneficial to all.

B5.3.1 Gifted, creative and highly sensitive children.pdf.

B5.4 The Gifted Adult. Lynn speaks to her experiences as an gifted person and describes the unique challenges faced by people with high IQ. The gifted's propensity towards perfectionism and their attraction to extremes should be carefully managed to ensure their talents have a life-enhancing effect.

B5.4.1 The Gifted Adult.pdf.

B5.5 The videos deal with the derivation of the Euclidean algorithm to determine the greatest common divisor. They are based on the fact that a natural number can be

represented uniquely by division with remainder. Then the algorithm is derived successively and applied interactively by tasks of exercises. It serves to promote algorithmic and number-theoretical thinking and requires the handling of numbers, operations, arithmetic laws and divisibility. This sequence is aimed at students up to the age of 14 years. The material and the videos are prepared in German language.

B5.5.1 Euclidean Algorithm.pdf

B5.5.2 Euclidean Algorithm2-video.pdf

B5.5.3 Euclidean Algorithm3-video.pdf

B5.5.4 Euclidean Algorithm4.video.pdf