



Co-funded by the
Erasmus+ Programme
of the European Union

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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

Reference Number: 2019-1-DE03-KA201- 059604

*The following Template is a guiding format for developing a Lesson Plan in a situation of supporting the students working in the context of the INNOMATH project in order to learn a **new mathematical topic**. This mathematical content is expected to be useful for the students in their effort for solving industrial problems.*

Learning Plan

Topic: Digital Geometry Software (DGS)

Target Group:

"Gifted" students in a high school at grade level 7-12 (secondary school)

Mathematical background of the students:

Basic geometry,

Ability to think algorithmically,

Goal/ Content/ Description:

Geometry is paramount in mathematical thinking. Illustrating, laying out graphically, helps understand a problem, posing the right questions, conjecturing and even proving. It makes sense to use software such as interactive digital geometry software, as these can quickly and clearly illustrate a figure which would be too time-consuming when drawing "by hand" and moreover much more versatile. This opens up venues for a lot of creative activities, see what was before or what will be after a very long process, changing the parameters to see different geometric effects.

The aim of this lesson is for students to develop digital skills applied to geometry. The aim of this lesson is to enable students to understand that geometric objects such as polygons, pictures can be defined in an interrelated parametric way... In doing so, an algorithmic way of thinking is to be developed that enables students to work on and present future problems using geometry. In this context, the various possibilities of graphical representation of data should be learned and used.

Objectives:

General Mathematical Objectives

- To develop skills for problem solving
- To develop motives and positive affective tendencies for mathematics
- To identify/ develop/ create applications of the related concepts and processes in the real world
- To develop digital skills/ through the use/ exploitation of digital means as help/ support in representations

- To develop the ability to think geometrically and algorithmically so that mathematical procedures can be transferred and translated into geometric figures
- To develop skills for collecting and modeling data and other information as they appear in the real world

Particular Mathematical Objectives

- Describe mathematical models hidden in geometric figures
- Describe explicitly a figure and relate it to a mathematical model
- Transfer mathematical models and their transformations into DGS commands
- Compare the effort required to solve problems by hand" and implementations in spreadsheets
- Describe the possibilities of spreadsheets
- Presentation of clear and beautiful problem solutions

Materials/ Tools:

- Computer
- Digital Geometry Software ([Geogebra](#), but [Cinderella](#), [Cabri](#), [Geometer's Sketchpad](#), [DGPAd](#) will do)
- Worksheet
- possible beamer

Resources used by the teacher:

Printouts of worksheets and interesting geometrical figures.

Resources for the student:

Articles, examples, exercises, ppt presentations, YouTube videos (eg [GeoGebra channel](#)) on the subject of GeoGebra by using the Internet.

For this the teacher is to prepare a list of webpages in the mother language of the students.

School Textbooks covering the topic.

Work sheets prepared by the teacher (eg an example can be found in the appendix)

Approaches/ Methodology:

This project-based approach to problem solving is used to clearly demonstrate to students the advantages of using a spreadsheet as opposed to doing the calculations "by hand". They should first do the calculations themselves before letting the spreadsheet do them in a very short time. In addition, they should acquire the ability not only to get to know other functions of the spreadsheet, but also to use them appropriately for future problems or projects.

Activities Plan:

Introductory activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
one week before	organise a room with enough computers with DGS (Geogebra) or make sure that students have their own laptop ready	

15 min	Let the student browse to https://www.geogebra.org/m/mgpnrkfq and try to discover how the mysterious triangle is built. They have to build other objects, the triangle ABC, some lines... (symmetries with respect to sides of the triangle)	Discussion on the issues Identify the important concepts Determine: Tools and types of objects
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Development and practicing activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
30 min	The students create an account to save their production. analyze a figure exhibiting an interesting pattern, such as an Op'Art production or an optical illusion. If necessary, the concepts and underlying calculation options must be repeated with the students.	Use the worksheet in the appendix. Use the concepts of dilation, translation, rotation, reflection, shearing, stretching, coordinates, barycenter...
50 min	The students create their own figures, looking for interesting geometric patterns and esthetical effects.	Bring printed examples from the Op'Art movement or other
10 min	The students discuss the advantages that a spreadsheet offers over production "by hand".	Discussion

Assessment activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Presentation and discussion of the production in a curation for an exhibition.	Organize a curation and exhibition of the productions

Reflection and Closure

What are the basic assumptions we have made in our mathematical models (mathematical perspective)?

What are the advantages and disadvantages of using spreadsheets (digital perspective)?

Assignment for further work

Use the Internet to optimise the presentations and get to know other spreadsheet tools, other generative art movements than Op'Art, other ways to produce this kind of artwork (other programming languages).

Appendix

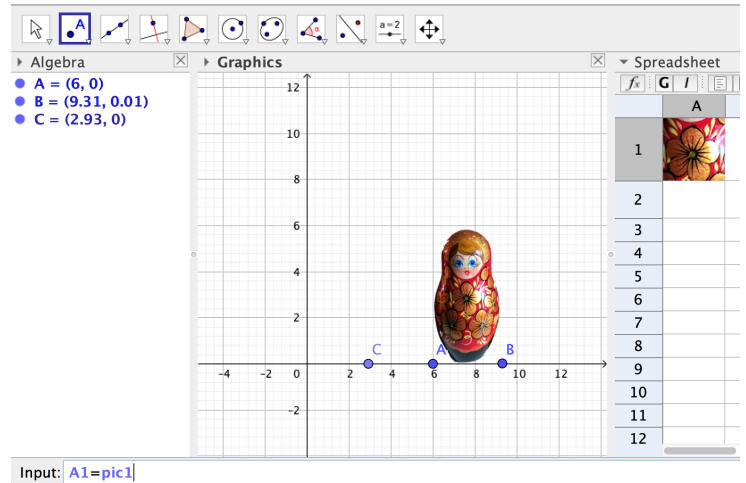
Matriochka activity

A picture is input into geogebra, this creates 2 points A and B. Create a 3rd point C. Then, enter the following cells in the Spreadsheet using the Input Bar:

A1=pic1

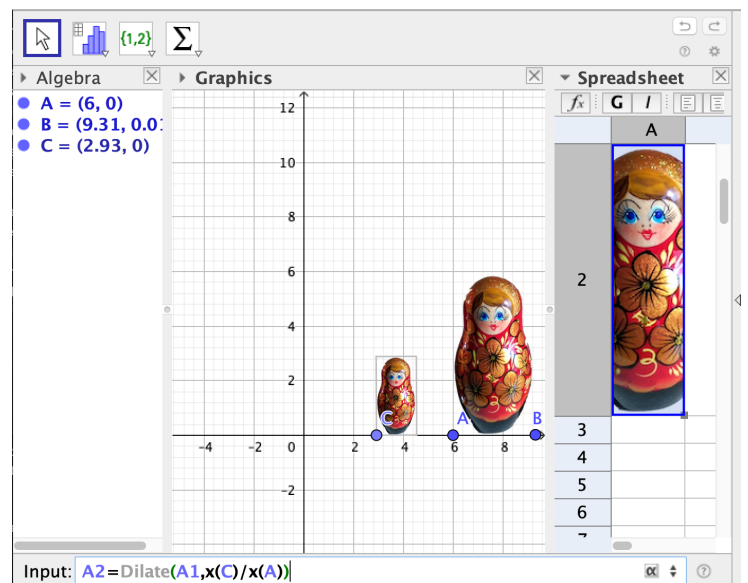
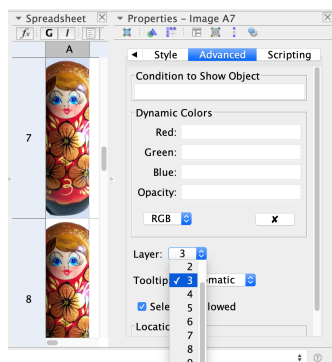
A2=Dilate(A1, x(C)/x(A))

This creates a second (smaller) picture.



Drag this formula from A2 to A10.

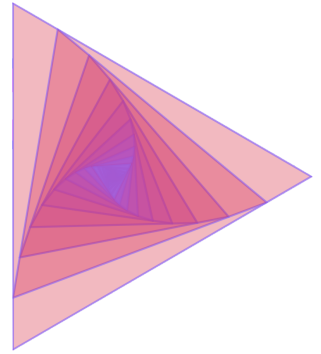
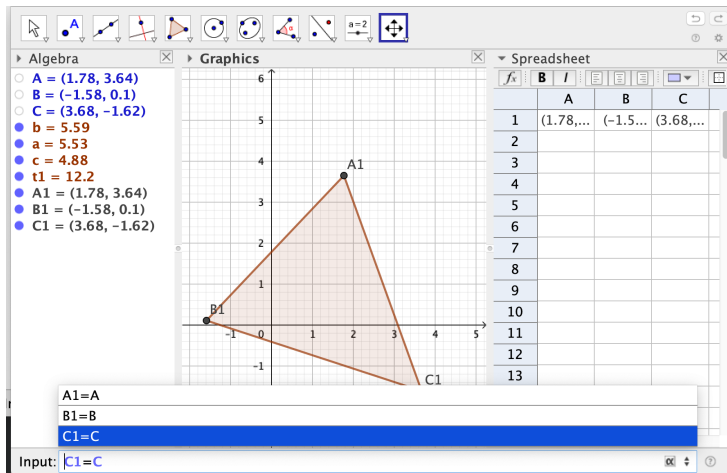
Play with the points. Beware of the layers that manage how the pictures are in front of one another. You find this in the Properties of a cell by right-clicking on it.



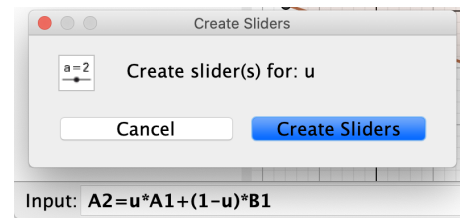
In fact, having A, B, C being complex numbers (redefining A=1+i for example) can produce much more interesting spiraling pictures, simply by addition and multiplication.

The spiral triangle

Create 3 points A, B, C and put the points in the spreadsheet:
A1=A, B1=B, C1=C and the triangle D1=Polygon(A1,B1,C1).



Then define the barycenter $A2 = u \cdot A1 + (1-u) \cdot B1$. This will define a slider u that you have to restrict between (say) -1 to 2.

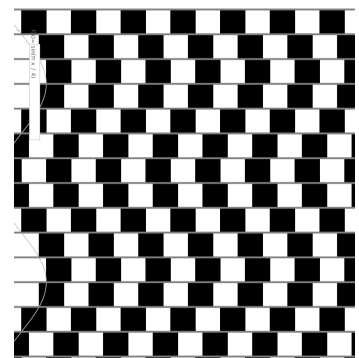
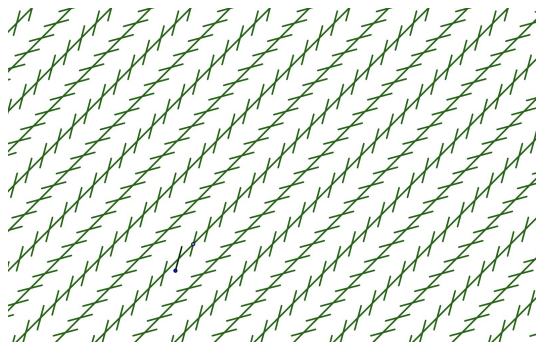
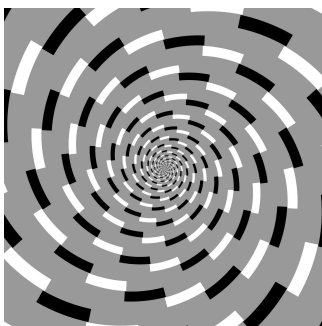


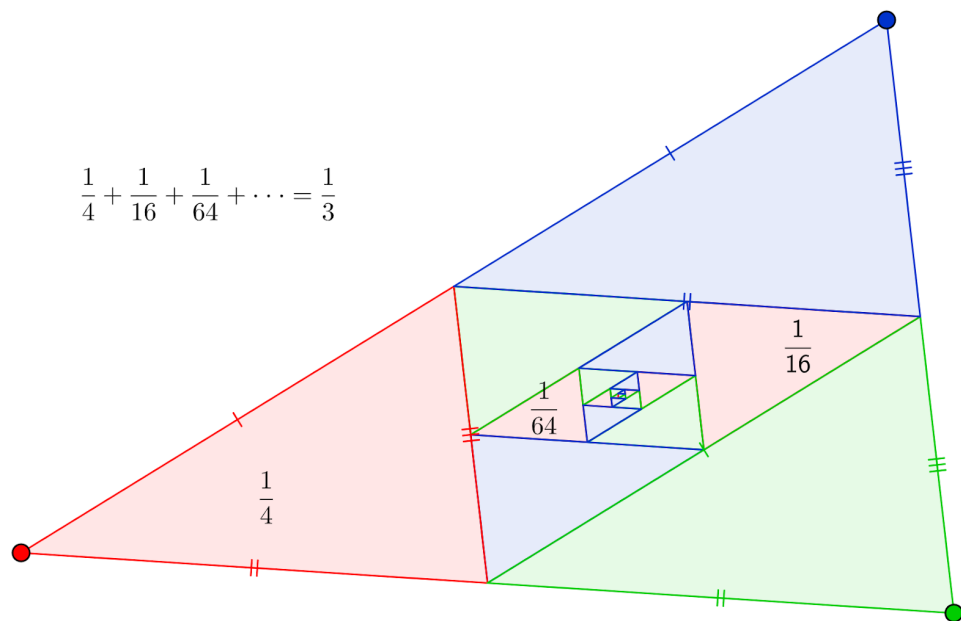
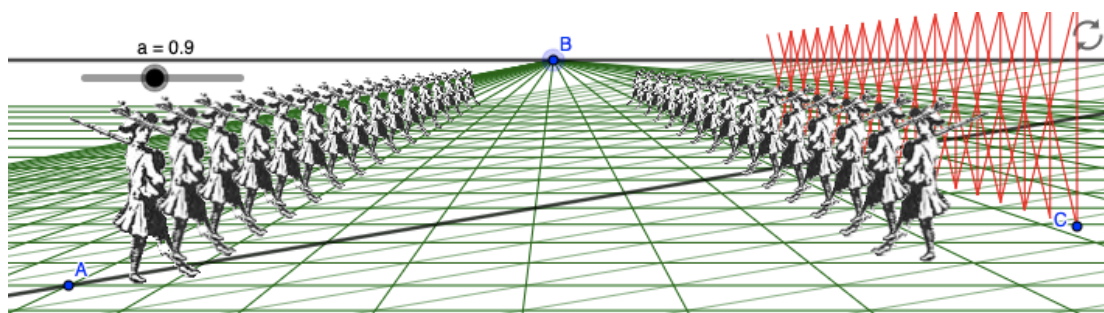
Then drag this A2 to B2 and C2. But modify C2 to $C2 = u \cdot C1 + (1-u) \cdot A1$ instead of $C2 = u \cdot C1 + (1-u) \cdot D1$ because D1 is a polygon. Then drag D1 into D2 to create the second triangle that should now read $D2 = \text{Polygon}(A2, B2, C2)$. Then, drag all 4 cells A2:D2 till the 10th line A2:D10 to complete the figure.

Modelling

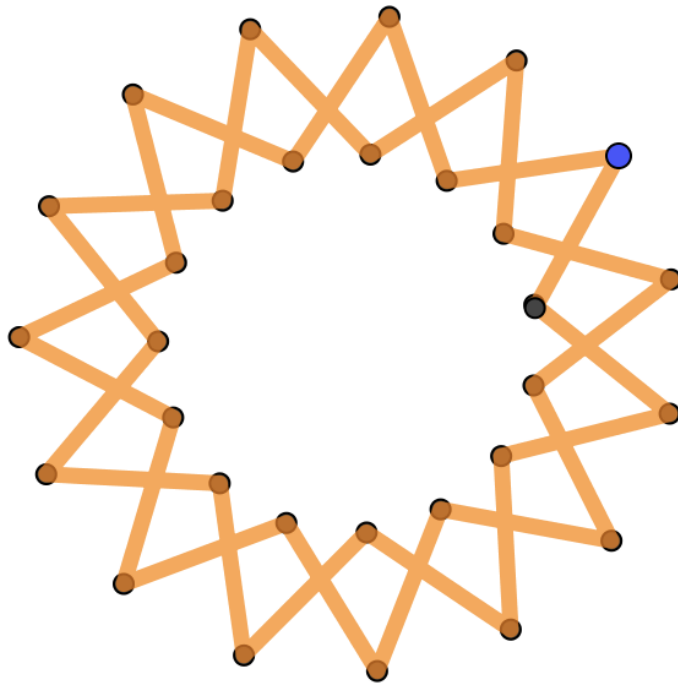
Print out some of the figures here and ask students to model them, identify the associated mathematical transformations and then implement them using the geometrical spreadsheet.

[Optical illusions list](#)

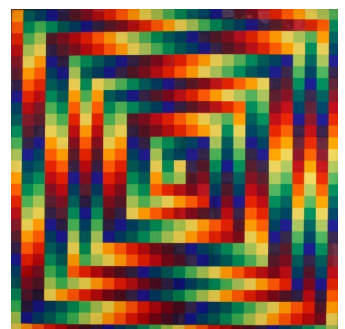
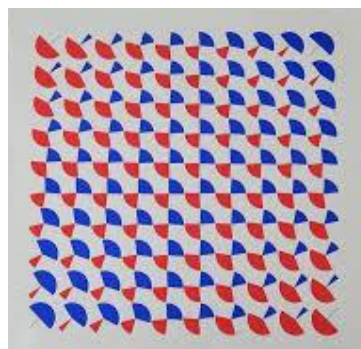
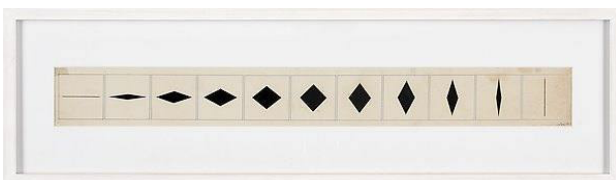
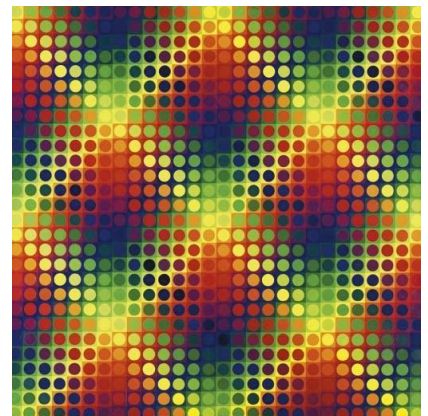
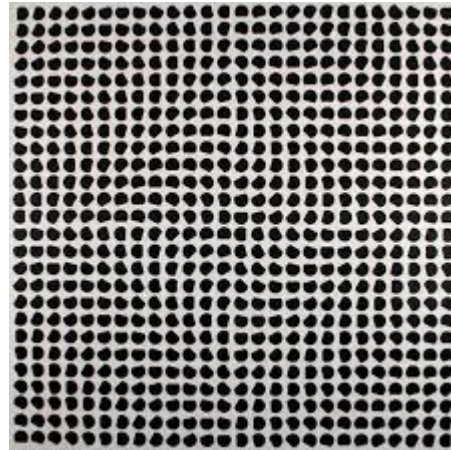




$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots = \frac{1}{3}$$



Julio Le Parc



Sol LeWit

