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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

Reference Number: 2019-1-DE03-KA201- 059604

*The following Template is a guiding format for developing a Lesson Plan in a situation of supporting the students working in the context of the INNOMATH project in order to learn a **new mathematical topic**. This mathematical content is expected to be useful for the students in their effort for solving industrial problems.*

Learning Plan

Topic: Spreadsheet geometric programming

Target Group:

"Gifted" students in a high school at grade level 7-12 (secondary school)

Mathematical background of the students:
Ability to use basic spreadsheet software and digital geometry software,
Ability to think algorithmically,
Ability to implement mathematical concepts in spreadsheets,

Goal/ Content/ Description:

In a lot of situations, some geometric objects are laid out in a pattern. It makes sense to use software such as spreadsheets, as these can quickly and clearly show the pattern, which would be too time-consuming when calculating "by hand". This opens up venues for a lot of creative activities, see what was before or what will be after a very long process, changing the parameters to see different geometric effects.

The aim of this lesson is for students to develop digital skills applied to geometric patterns. The aim of this lesson is to enable students to understand that functions can act not only on numbers, but as well on geometric objects such as polygons, pictures... In doing so, an algorithmic way of thinking is to be developed that enables students to work on and present future problems using spreadsheet programming. In this context, the various possibilities of graphical representation of data should be learned and used.

Objectives:

General Mathematical Objectives

- To develop skills for problem solving
- To develop motives and positive affective tendencies for mathematics
- To identify/ develop/ create applications of the related concepts and processes in the real world
- To develop digital skills/ through the use/ exploitation of digital means as help/ support in calculations and representations

- To develop the ability to think algorithmically so that mathematical procedures can be transferred and translated into geometric spreadsheets
- To develop skills for collecting and analyzing data and other information as they appear in the real world

Particular Mathematical Objectives

- Describe mathematical models hidden in geometric figures
- Describe explicitly a figure and relate it to a mathematical model
- Transfer mathematical models and their transformations into spreadsheet commands
- Compare the effort required to solve problems by hand" and implementations in spreadsheets
- Describe the possibilities of spreadsheets
- Presentation of clear and beautiful problem solutions

Materials/ Tools:

- Computer and/or scientific calculator
- Spreadsheet (Geogebra)
- Worksheet
- possible beamer
- possibly presentations

Resources used by the teacher:

Introductory books on the use of spreadsheets and digital geometry software.
Printouts of Op'Art and interesting geometrical figures.

Resources for the student:

Articles, examples, exercises, ppt presentations, YouTube videos
For this the teacher is to prepare a list of webpages in the mother language of the students.
School Textbooks covering the topic.
Half baked worksheets prepared by the teacher (eg an example can be found in the appendix)

Approaches/ Methodology:

This project-based approach to problem solving is used to clearly demonstrate to students the advantages of using a spreadsheet as opposed to doing the calculations "by hand". They should first do the calculations themselves before letting the spreadsheet do them in a very short time. In addition, they should acquire the ability not only to get to know other functions of the spreadsheet, but also to use them appropriately for future problems or projects.

Activities Plan:

Introductory activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments

one week before	organise a room with enough computers with geometric spreadsheet software (Geogebra) or make sure that students have their own laptop ready	
15 min	Realize together the Russian matriochka activity: 	Discussion on the issues Identify the important concepts **Determine:** Variables and parameters

Development and practicing activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
30 min	The students analyze a figure exhibiting an interesting pattern, such as an Op'Art production or an optical illusion. If necessary, the concepts and underlying calculation options must be repeated with the students.	Use the worksheet in the appendix. Use the concepts of dilation, translation, rotation, reflection, shearing, stretching, coordinates, barycenter...
50 min	The students create their own figures, looking for interesting geometric patterns and esthetical effects.	Bring printed examples from the Op'Art movement or other
10 min	The students discuss the advantages that a spreadsheet offers over production "by hand".	Discussion

Assessment activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Presentation and discussion of the production in a curation for an exhibition.	Organize a curation and exhibition of the productions

Reflection and Closure

What are the basic assumptions we have made in our mathematical models (mathematical perspective)?

What are the advantages and disadvantages of using spreadsheets (digital perspective)?

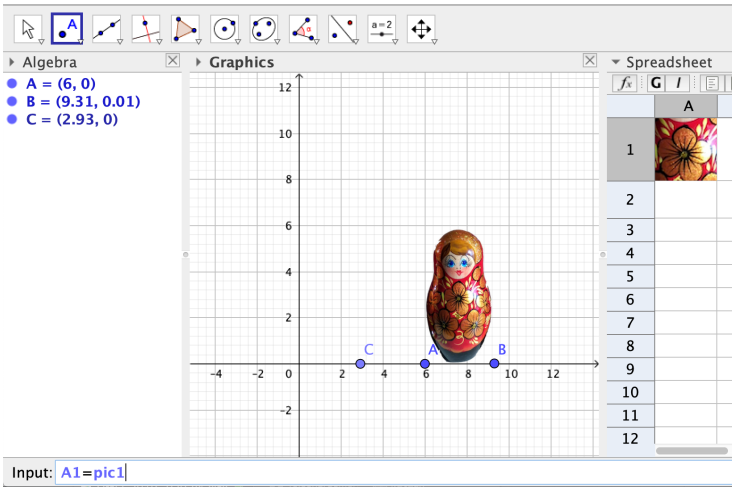
Assignment for further work

Use the Internet to optimise the presentations and get to know other spreadsheet tools, other generative art movements than Op'Art, other ways to produce this kind of artwork (other programming languages).

Appendix

Matriochka activity

A picture is input into geogebra, this creates 2 points A and B. Create a 3rd point C. Then, enter the following cells in the SpreadSheet using the Input Bar:



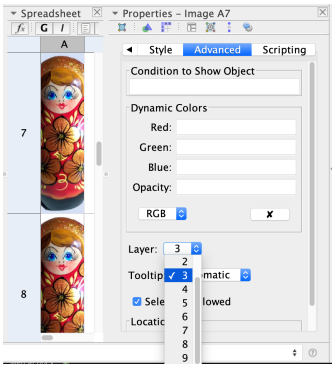
A1=pic1

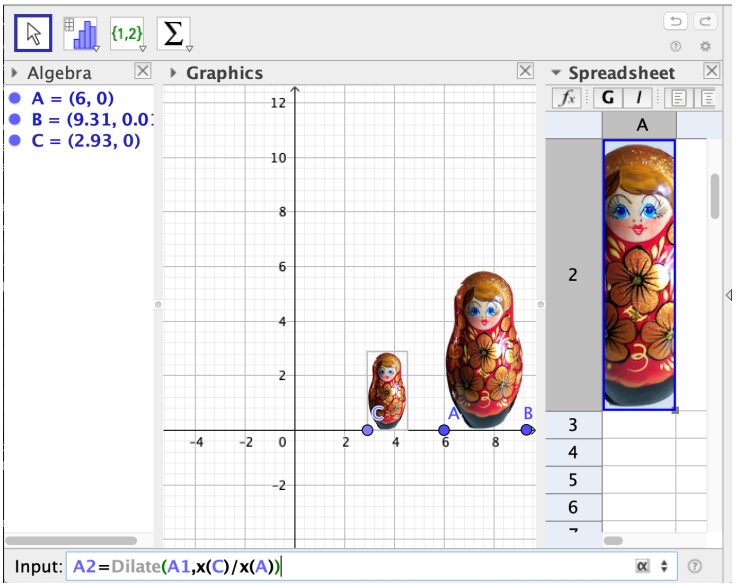
A2=Dilate(A1, x(C)/x(A))

This creates a second (smaller) picture.

Drag this formula from A2 to A10.

Play with the points. Beware of the layers that manage how the pictures are in front of one another. You find this in the Properties of a cell by right-clicking on it.

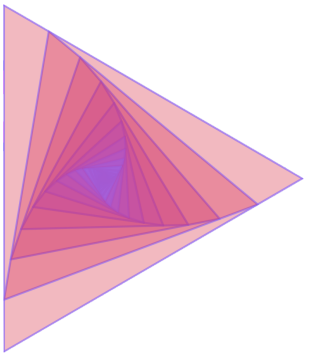


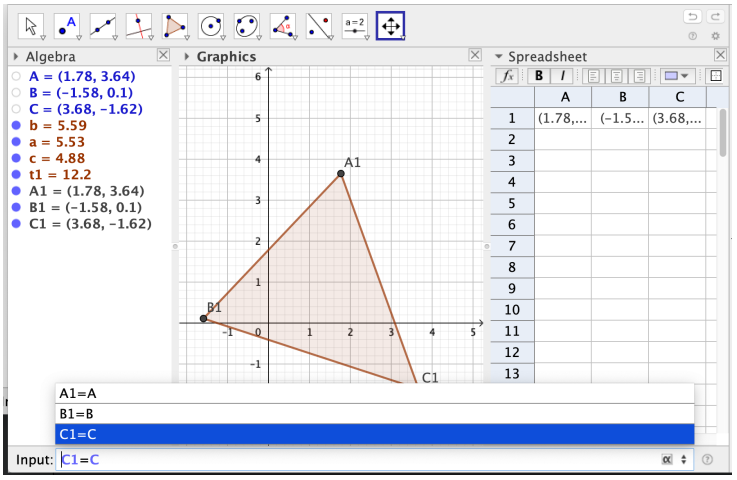


In fact, having A, B, C being complex numbers (redefining A=1+i for example) can produce much more interesting spiraling pictures, simply by addition and multiplication.

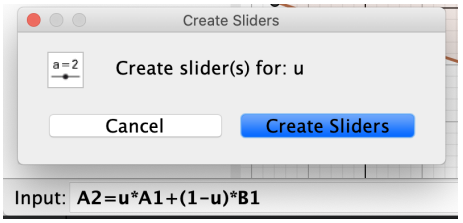
The spiral triangle

Create 3 points A, B, C and put the points in the spreadsheet: A1=A, B1=B, C1=C and the triangle D1=Polygon(A1,B1,C1).





Then define the barycenter A2=u*A1+(1-u)*B1. This will define a slider u that you have to restrict between (say) -1 to 2.

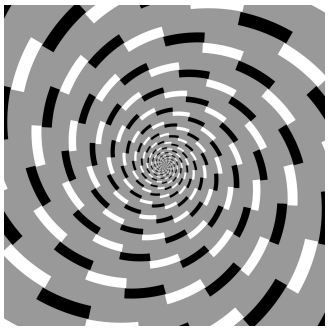


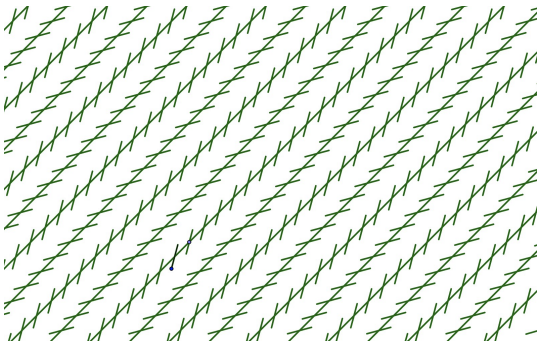
Then drag this A2 to B2 and C2. But modify C2 to C2=u*C1 + (1 - u)*A1 instead of C2=u*C1 + (1 - u)*D1 because D1 is a polygon. Then drag D1 into D2 to create the second triangle that should now read D2=Polygon(A2,B2,C2). Then, drag all 4 cells A2:D2 till the 10th line A2:D10 to complete the figure.

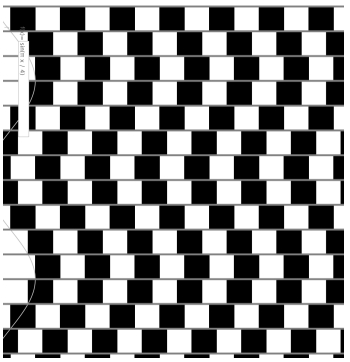
Modelling

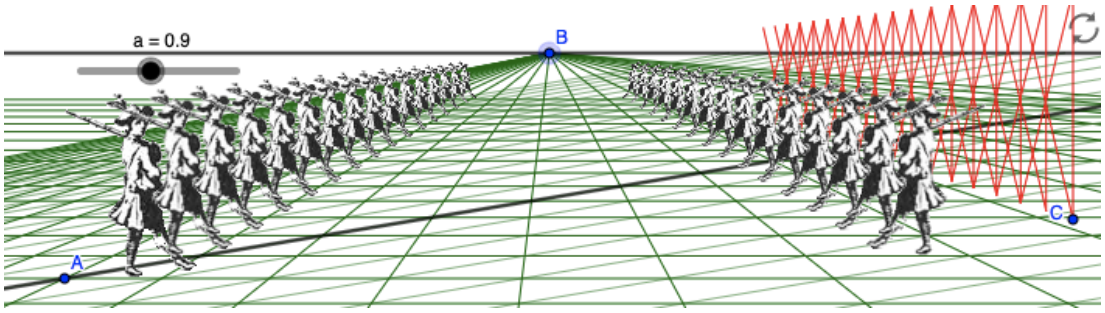
Print out some of the figures here and ask students to model them, identify the associated mathematical transformations and then implement them using the geometrical spreadsheet.

Optical illusions list

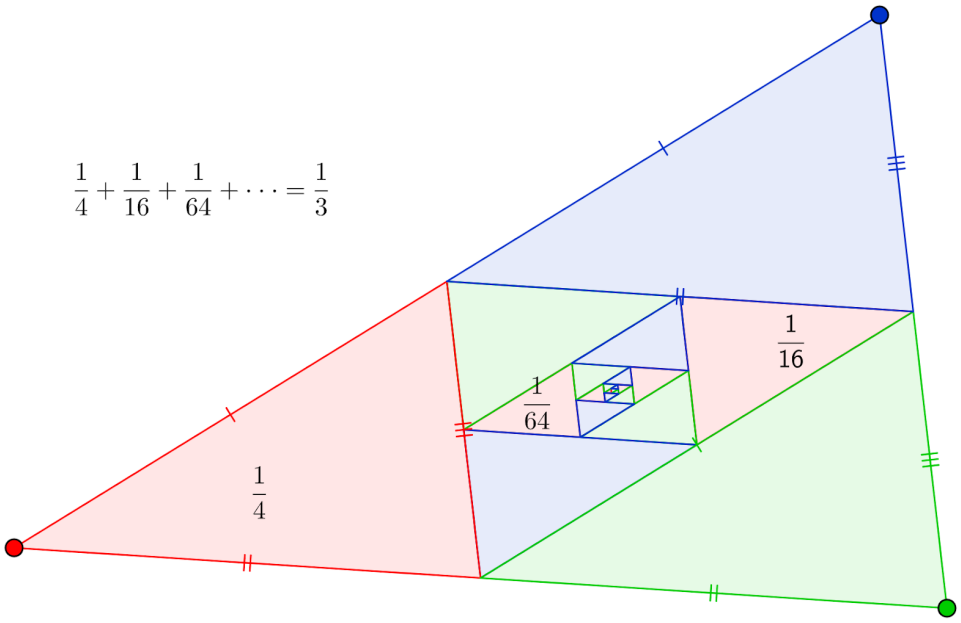


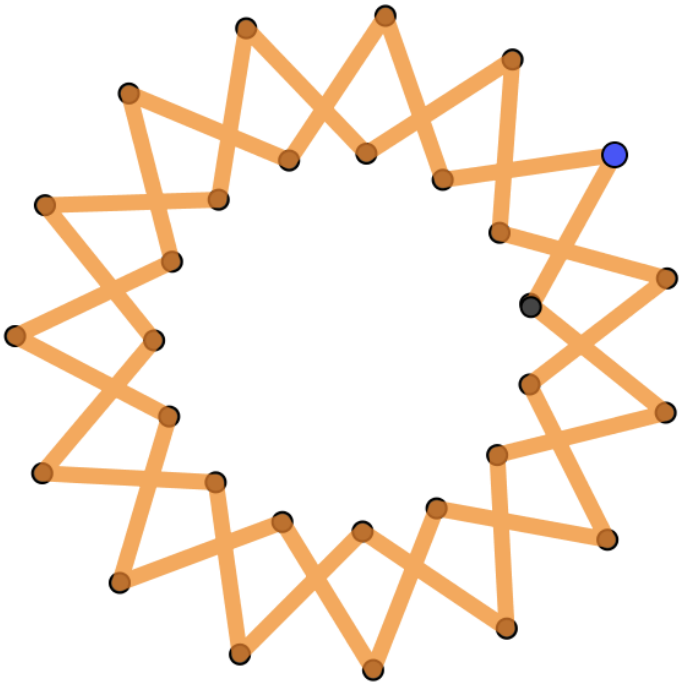




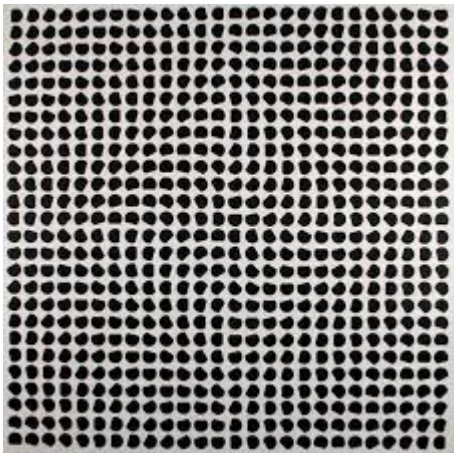


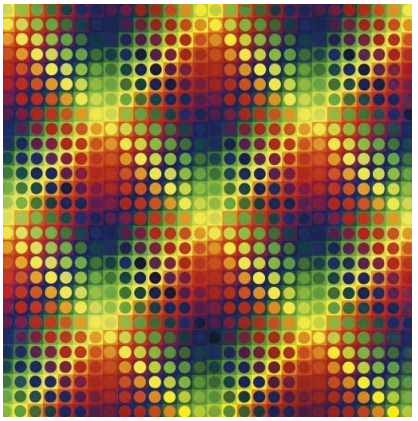
$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots=\frac{1}{3}$$

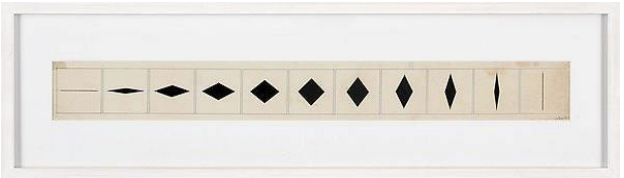


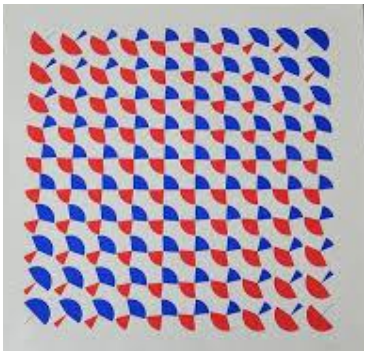


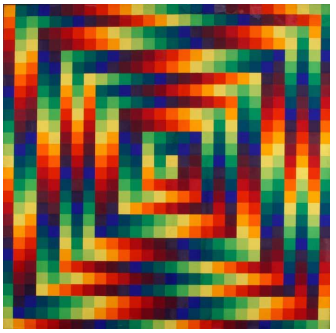
Julio Le Parc











Sol LeWit