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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

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Learning Plan

Topic: Percentages and Probabilities

Target Group: "Gifted" students to a high school (Y7-12)

Goal/ Content/ Description: The main goal of any normal business is to maximise profits — nobody is getting up and going to work with the object of losing money. As such, business problems around profitability are the everyday work for management consultants. One goal in the industry world is make and at the same time to gain the best deal. Always people are concerned if it is worth to risk and invest some money. They are interested to know the maximum possible profit or the minimum possible loss.

The goal of this learning plan is to help students understand how to use percentages and probability knowledge to find the highest expected profit or least expected loss. In addition, it will give the essential skills to students to solve more complicated and advanced industrial problems.

Objectives:

To develop skills to analyse the data of the real world problems.

To develop skills to structure their approach to solve the real world problems.

To develop skills to understand the problems.

To develop mathematical skills to solve similar or more advanced real world problems.

Materials/ Tools:

Scientific calculators

Worksheet

Internet network

Resources used by the teacher:

Article examples, exercises on the topic by using the Internet.

School mathematics textbooks covering the topic

The example used in Appendix 1 found in the website:

https://www.myconsultingcoach.com/management-consulting-cases/case/dog-microchips

Appendix 1 found in the website:

https://www.youtube.com/watch?v=WsnBNjXP0CM

Resources for the student:

Article examples, exercises on the topic by using the Internet.

School textbooks covering the topic

Worksheet prepared by the teacher.

Approaches/ Methodology:

Activities Plan:

Introductory activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
Previous day	Revise the concepts of solving linear equations, percentages, proportions and basic probabilities. Study the definitions of the economic terms	See Appendix 1 See Appendix 2
15 min	A furniture company sells sofas. There is one specific model which costs $500 and the company sells this sofa with 65% profit. The chance of persuading the client to buy this sofa is 15%. If they upgrade the sofa design and pay $40 each month for advertisement, the probability that a client will buy this sofa will increase to 35%. However, this upgrade on sofa's design will increase the sofa's cost by 20% while the selling price will remain the same. The shop has on average 5 clients monthly. Please advise the company if it is worth to make this upgrade on sofa's design and find out the maximum expected profit.	Discussion on the real world problem. Discussion on the meaning expected profit. Discussion on the difference between the real profit and the expected calculated profit.

Development activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	The students should spend some time to structure their approach on how to solve the real world problem.	Possible Approach: 1.Profit= Revenues – Costs 2.Revenue= Product quantity x Price 3.Identify the product quantity 4.Estimating Revenue 5.Estimating costs 6.Estimating profit 7. Deciding whether to invest in a design change 8. Final recommendation
15 min	Investigate the solution	See Appendix 3. Discussion:

		The role of probability in this real world problem.

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
20 min	Provide a set of exercises for practise from a given handout.	Discussion on problems. Provide solutions.

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
40 min	Give a problem and ask for the solution in the classroom.	See Appendix 4. Provide solutions of the assessment.

Reflection and Closure

After this lesson, do the students feel confident and ready to solve such real world problems?

What are the advantages and disadvantages of the final recommended solution to such real problems?

Assignment for further work

Using the internet, find similar or more advanced real world problems and try to solve them and state your recommendation for the maximum profit of a company.

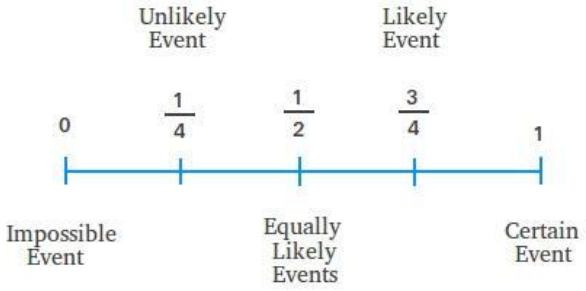
Appendix 1

Probability

Definition: the likelihood of an event occurring.

Probability line

The probability line is a line that shows probabilities and how these probabilities relate to each other.



Probability line

Since the probability of an event is a number from 0 to 1, we can use the probability line above to show the possible ranges of probability values.

Expected values

Definition: The average outcome we expect if we run an experiment many times.

The basic expected value *formula* is the probability **of an event multiplied by the amount of times the event happens:** $E(X) = P(X) \times n$

Example:

Find out how many times you will get a spade if you draw a card from a pack of 52 playing cards 20 times.

$P(\ spade) = \frac{13}{52} = \frac{1}{4}$

$E(spade) = \frac{1}{4} \times 20 = 5.$

An expected value of 5 means we expect to get a spade 5 times if we draw a card from a pack of 52 playing cards 20 times.

However, *nothing guarantees* us getting a spade EXACTLY 5 times. Realistically, we could get a spade 4 times, 6 times or even 20 times.

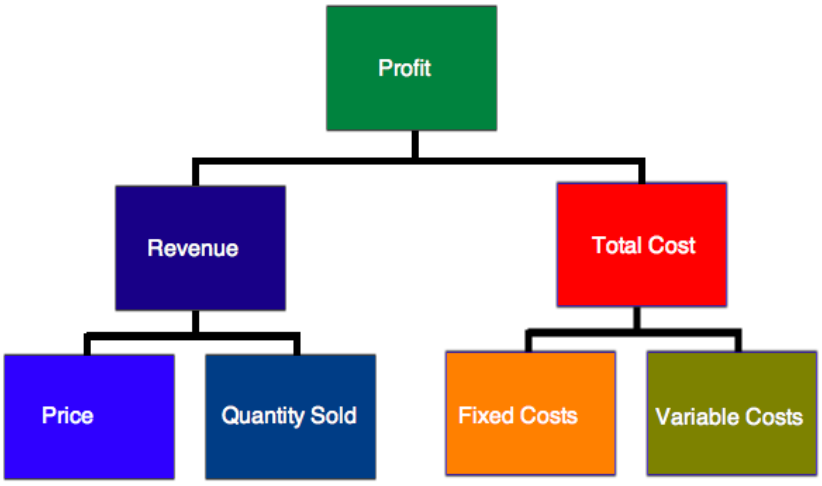
Appendix 2

Definitions of the economic terms

- Firms incur expenses when they produce goods and services. These expenses are called costs.
- These costs are classified by economists according to how they behave when output changes.

- Fixed Costs **(FC)** costs that do not vary with the level of output. For example, rent and insurance.
- Variable Costs**: (VC)** costs that change when output levels change. For example, raw materials and fuels.

- Total Costs **(TC)**: Fixed and Variable Costs added together **TC=FC+VC**
- Revenue: The amount of money a firm receives from selling its output is called total revenue. Total Revenue **(TR)**= Product quantity x Price
- Profit: The difference between total revenue and total costs. **Profit=TR-TC**
- If Total Costs exceed Total Revenue, then a Loss is made!



Appendix 3

A furniture company sells sofas. There is one specific model which costs $500 and the company sells this sofa with 65% profit. The chance of persuading the client to buy this sofa is 15%. If they upgrade the sofa design and pay $40 each month for advertisement, the probability that a client will buy this sofa will increase to 35%. However, this upgrade on sofa's design will increase the sofa's cost by 20% while the selling price will remain the same. The shop has on average 5 clients monthly. Please advise the company if it is worth to make this upgrade on sofa's design and find out the maximum expected profit.

Solution:

Estimating Revenue

$Selling\ price\ = 500 \times 1.65\ = \$825$

$Revenues\ =\ Product\ quantity \times Selling\ price$

$Revenues\ =\ 825 \times 5\ =\ \$4125$

$Costs\ without\ upgrading\ and\ advertisement\ =\ 500 \times 5\ =\ \$2500$

$Costs\ with\ upgrading\ and\ advertisement\ =\ 500 \times 1.20 \times 5\ =\ \$3000$

Profit

Profit= Revenues – Costs

$Profit\ without\ upgrading\ and\ advertisement\ =\ 4125\ -\ 2500\ =\ \$1625$

$Profit\ with\ upgrading\ and\ advertisement\ =\ 4125\ -\ 3000\ =\ \$1125$

Deciding Whether to Invest in $upgrading\ and\ in\ advertisement$

$Expected\ profit\ without\ upgrading\ and\ advertisement\ =\ 1625 \times 0.15\ =\ \$243.75$

$Expected\ profit\ with\ the\ upgrading\ and\ advertisement\ =$

$(1125 \times 0.35\ -\ 40) + (-\ 40 \times 0.65) = \$327.75$

Final Recommendation

The expected profit after upgrading is higher. Hence, it is worth the company to make this upgrade on the sofa design and pay for advertisement and the maximum expected profit will be $\$327.75$.

Appendix 4

All dogs licenced in the UK must already be fitted with a subcutaneous microchip which identifies the dog with its owner's contact details. However, the UK government is bringing forward legislation which would require all newly-licensed dogs to be fitted with a new kind of microchip also containing a passive transponder. This would allow lost dogs to be located.

If the legislation is passed, the government will have to issue a contract to manufacture these new microchips.

Stratos is a UK microchip manufacturer, making chips for a wide range of applications. Management at Stratos are interested in making these new subcutaneous location chips and has asked us to find out whether they should go forward with trying to secure the contract.

The following information are given:

- The total number of dogs in the UK is approximately 12m
- Average canine life expectancy is 8 years
- 94% of dogs are licenced
- The government is prepared to pay £15 each for the new chips
- Chips will cost Stratos £4 each to manufacture (with this including labour, raw materials and other costs).
- To produce these additional microchips alongside their current contracts, Stratos will have to increase the size of their production facilities.
- Stratos can lease a unit adjacent to their current production line for £750k per year.
- This new production line will also require new equipment to be purchased at a cost of £6m.
- A few different location technologies can be used to make chips that are compliant with the new legislation. However, all are proprietary and will require both an initial lump payment plus an additional licencing payment for each chip manufactured.
- Stratos has two options to licence locator technology:

  Cartograph

  Will require a lump payment of £500k and per-unit licencing fee of £1

  Transpose

  Will require a lump payment of 250k and a per-unit licencing fee of £2

- Stratos has four other competitors which are capable of bidding for the government contract.
- All have roughly equivalent market share in this space and all have an equivalent chance (approximately 20%) of winning the government contract.
- However, if Stratos invests £1m to lobby government ministers, they believe that they can raise their chances of securing the contract to as much as 75%.

Solution:

Estimating Volume

Thus, we have a value for the price (£15), but must still estimate the number of chips required.

The student might make this estimation via multiple different methods. Here, we will use the replacement concept.

To begin, we assume that the number of dogs in the UK has remained constant.

To maintain stable numbers, the number of new dogs each year must equal the number which die.

We calculate this number as follows:

Replacement Rate = Population/Lifespan = 12m/8 = 1.5m new dogs per year

94% of these dogs will be licenced in accordance with the law

1.5 x 0.94 = 1.41m new licences per year

We can round this to 1.4m new chips required per year from the start of the programme

Note that dogs which have already been licenced are not covered by the new legislation, so need not be considered.

Estimating Revenue

We can now plug our values for volume and price into the equation above.

Revenue = Product quantity x Price

Revenue = 1.4m x 15

Revenue = £21m

Production Costs

Cost to manufacture = number of chips x unit cost

Cost to manufacture = 1.4m x 4 = £5.6m per annum

Cost of new facility = rental cost + equipment cost

Cost of new facility = 750k + 5m = £6.75m in year one

Total production costs = 5.6m + 6.75m = 12.35m

Licencing Costs

These will need to calculated for both possible providers:

Cartograph

Cost to licence tech = lump sum + (unit fee x number of units)

Cost to licence tech = 500k + (1 x 1.4m) = £1.9m

Transpose

Cost to licence tech = lump sum + (unit fee x number of units)

Cost to licence tech = 250k + (2 x 1.4m) = £3.05m

The student should note that, at the volumes required here, Cartograph is the cheaper option and thus the one that should be engaged if production goes ahead.

Total Costs in Year One

Total Costs = production costs + licencing costs

Total Costs = 12.35m + 1.9m = 14.25

Profit

Now that the student has values for both revenues and costs, they can calculate Stratos's projected profits for year one of the dog microchip contract.

Year One Profit = Revenues – Costs

Year One Profit = 21m – 14.25m

Year One Profit = £6.75m

Deciding Whether to Invest in Lobbying

In order to decide on the best course of action here, the student should compare the expected profits with and without lobbying.

Expected Profit Without Lobbying

(6.75 x 20%) = £1.35m

Expected Profit With Lobbying

(6.75 x 75% - 1) + (-1 x 25%) = £3.81m

Useful Note: Stratos has a chance of 25% not winning the contract, despite the fact that he has already paid the £1m for lobbying. Therefore, the 25% of £1m needs to be subtracted from the expected profit.

The expected profit with lobbying is higher. Thus, Stratos should invest in lobbying government ministers to help secure the deal.

Final Recommendation

The student should provide a brief, structured recommendation advising Stratos on how to proceed.

If the legislation requiring the new microchip type for dogs is passed, Stratos should attempt to win the government contract to provide these chips as the expected profits will be £3.81m in year one, with further profits in subsequent years.

In doing so, Stratos should licence locator technology from Cartograph, as this minimises licencing costs and so boosts profits.

They should also invest in lobbying government ministers to increase their chance of winning the contract rather than one of their competitors. This move boosts expected profits.

The student should also note any potential risks or other issues which might need to be considered before going forward.