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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

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The Tour

Learning Plan

Topic: algorithmization

Target Group: grades 10 – 12

Goal/ Content/ Description: The task will be treated as a mathematical problem, requiring the construction of a mathematical model of a situation known from everyday life. Prepare a diagram to calculate the cost of a one-day school trip. The diagram should be general and legible so that another organizer can use it when planning the same trip. Students must independently determine all the elements that make up the cost of a trip for one student, such as: means of transport, number of guardians, costs of tickets to the museum / theatre / cinema, insurance, various options for determining the cost of these attractions, e.g. different ticket prices for guardians and students, or no fees for guardians, but also adding additional costs - for example - fees for a guide or a cloakroom. The problem allows for a great deal of freedom, it enables creativity, allowing the scheme to be supplemented with its own additional assumptions, considered essential.

Objectives:

- taking over and assimilating mathematical information transmitted in various forms from various sources
- practicing basic elementary mathematical skills (algorithms, logical operations, semi-algorithms, geometric constructions)
- solving typical problems using basic mathematical methods and techniques
- editing, writing, illustrating with diagrams, coding, etc., mathematical content, training in using mathematical language in its various forms
- organizing and memorizing previously learned knowledge
- specifically creative activity going beyond the above-mentioned activities (noticing and formulating problems, constructing and defining new concepts for the learner, discovering, formulating and proving theorems, generalization and specification, using mathematics to solve problems in other fields)

Materials/ Tools: computer, internet network, blackboard, calculator, flipchart, program, Creately Diagrams, edrawsoft,

Resources used by the teacher: Articles, examples, exercises, webpages on the topic by using the Internet.

Resources for the student: Articles, examples, exercises, webpages on the topic by using the Internet.

Approaches/ Methodology: In the first phase, students must decide on the basic parameters of the trip: destination, means of transport, number of participants (students, guardians, visitors), attractions (cinema, theatre, amusement park, zoo, etc.), meals, insurance and more.

In the second phase, students have to collect data on the parameters established in the first phase, such as ticket prices, transport costs, meal prices, number of exemptions, discounts, insurance costs and more.

In the third phase, students are asked to analyse the collected data, and then use it to calculate the cost of participation in the trip for one student.

The fourth phase consists in departing from the previously considered data and preparing, on the basis of the previous stages, an algorithm allowing for the calculation of trip costs for new data.

The fifth phase consists in preparing a computer simulator of costs.

The role of the teacher is to present and explain the problem posed. In addition, his tasks will include: discussion and possible suggestions of factors that must be taken into account, assistance in collecting data and their analysis, preparing a graphic presentation of the results of their work with the students, assistance in the preparation of a computer version of the simulator.

Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
two days earlier/60 min	the teacher presents the students with the task and explains the principles of the project implementation; students reflect together on factors to be considered; students share tasks for collecting the necessary information	the teacher must make sure that students understand the problem and select the appropriate factors influencing the cost of the trip students share tasks, it is advisable that everyone in the assigned area should collect a wide range of data (e.g. consider the cost of travel by train, bus, rented coach) the teacher can provide tips on the sources of information sought
to the next meeting	students collect information	the group remains in touch, supports itself in its tasks if necessary, looks for alternative solutions

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
the day of meeting in class/30 minutes	students collect all the acquired data in one database, analyse various versions of the trip in the area of predetermined factors	
the day of meeting in class/30 minutes	Students prepare a diagram of the algorithm taking into account all the factors they choose, and then simulate various possibilities, collecting data in the form of the price of a trip for one student as a result of the work.	

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
the day of meeting in class/60 minutes	Find the roots of a quadratic equation. (Appendix 1) Students try to find and graphically present an algorithm for checking whether a given number is prime. (Appendix 2) The students try to find the algorithm and its visualization in the problem of "Cesar's code". (Appendix 3) Students try to find an algorithm for finding an approximate solution to a nonlinear algebraic equation and present it graphically. (Appendix 4)	The role of the teacher is to oversee the course of work, provide guidance and support students.

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
After lesson	Students evaluate their work. They wonder: have they considered all the possibilities of the issue? Diagnose to what extent the knowledge from everyday life turned out to be useful? Were the results of their work in line with expectations? What could be improved in the work of the group or in the operation of the algorithm? Will the tools they have developed prove useful to others?	

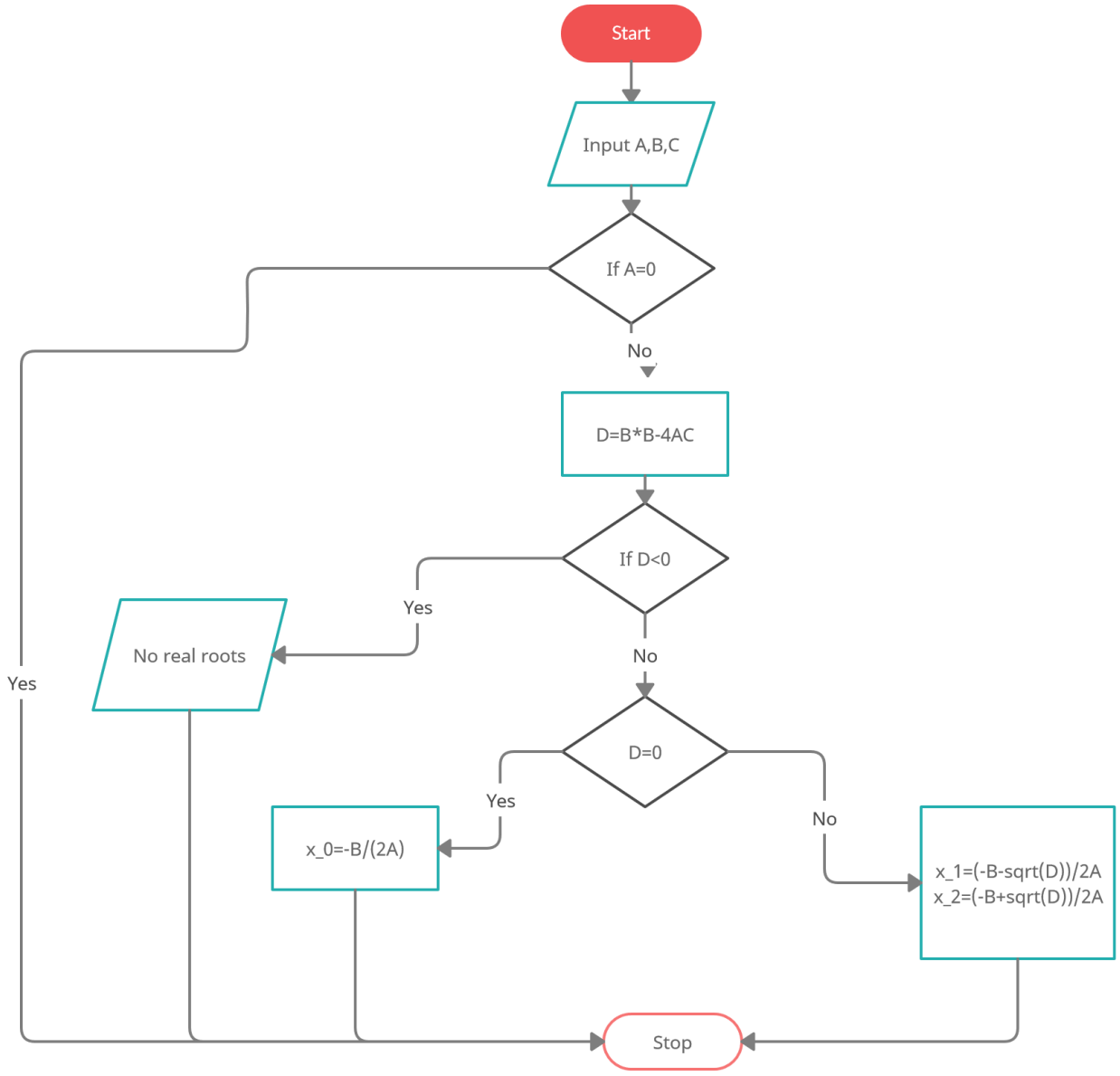
Reflection and Closure

Assignment for further work

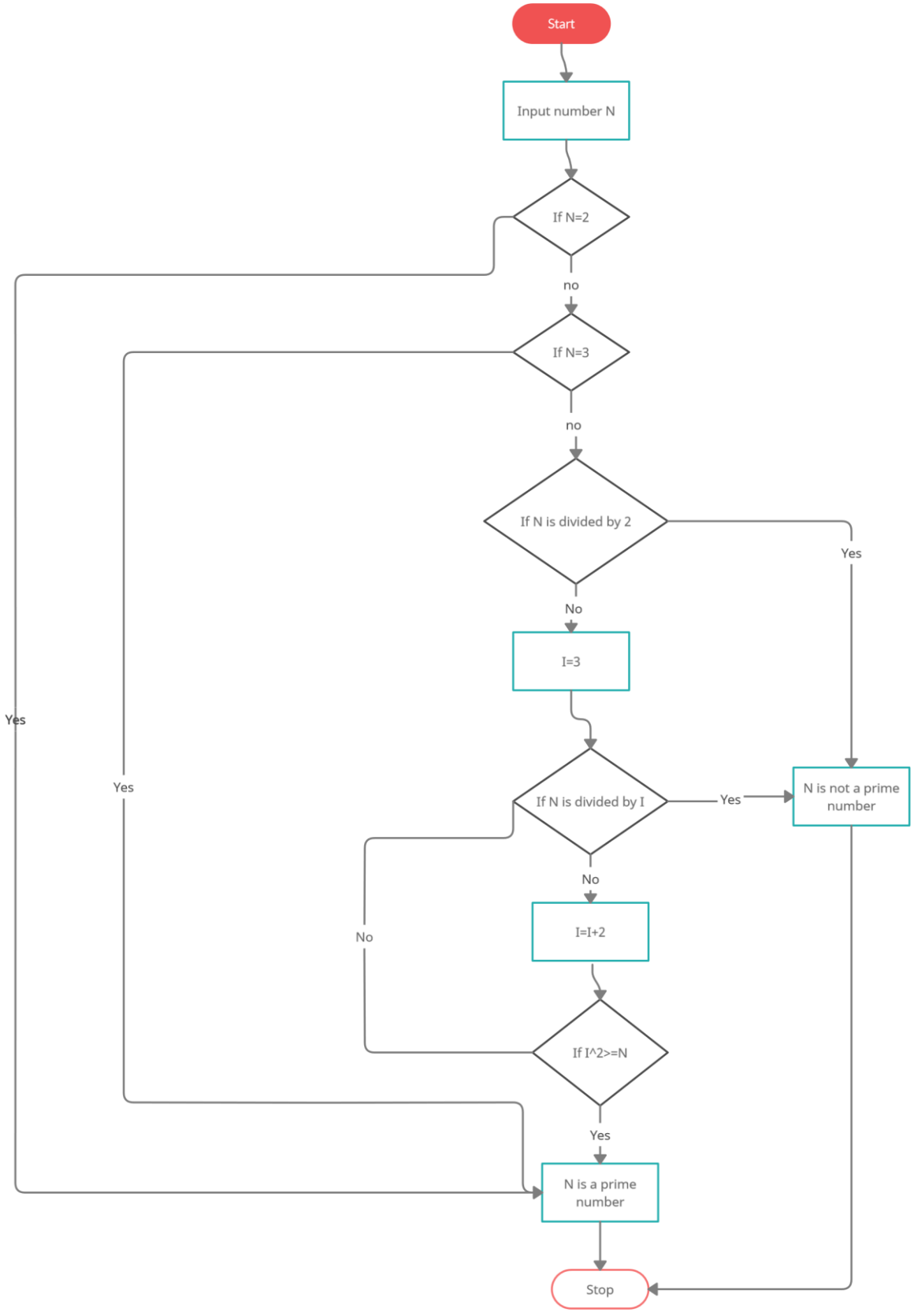
Finally, students are asked to look for mathematical problems that can be solved algorithmically, find an algorithm and present its visualization.

Appendix 1

Example: There is given a quadratic equation. We want to compute its roots.



Appendix 2 Visualisation of an algorithm to check whether a given number is a prime number.



Appendix 3

The text is encoded using the "Caesar Code" method. The method requires a scrambling word (Code), which must also be known during the decoding operation. For simplicity, the example shows only 26 capital letters of the English alphabet:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Coding

The method consists in adding the coded text position number and the code text position number. If the obtained sum exceeds the number of available characters (26), then 26 is subtracted from the sum. The resulting sum values define the coded text. The input data for encoding are: the text to be encoded and the text of the scrambling code.

Example:

	H	E	L	L	O	W	O	R	L	D	6	4	10	10	13	21	13		16	10	3
+	I	N	N	O	M	A	T	H	I	N	7	12	12	13	11	0	18		6	7	12
	O	R	Y	X	Y	W	G	X	S	Q	13	16	24	23	24	21	31(5)		22	17	15

Decoding

When decoding the code text position number should be subtracted from the coded text position number. If the value obtained is less than 0, then 26 should be added to it. The input data for decoding are: coded text and scrambling code text.

Appendix 4

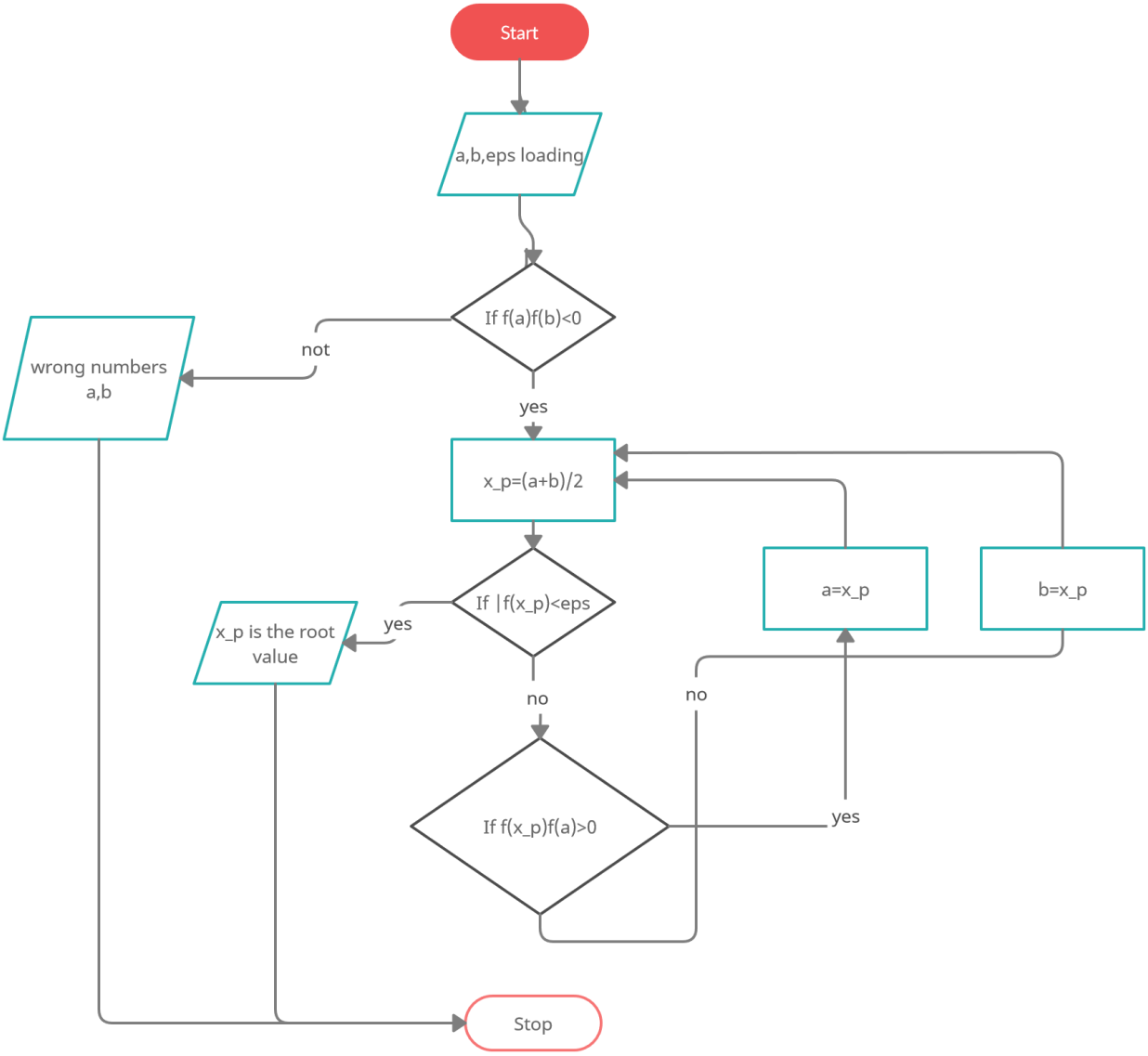
If we take the accuracy of calculations as *eps*, it is done according to the following algorithm:

1. Assume (arbitrarily) the interval $[a, b]$ and the accuracy eps
2. We check if $f\ (a)\ *\ f\ (b)\ <0$
3. The answer YES means that we have accepted the limits of the interval well and the root of the equation is inside the interval and we can continue our activities.
4. The answer NO means that the limits of the range were adopted incorrectly and the calculations should be interrupted.
5. Calculate (dividing the interval into half - hence the name "half interval method") the value of the expression $\frac{a+b}{2}$
6. If $|\ f\ (x_p)\ |\ <eps$ program termination, xp is the root value.
7. If $|\ f\ (x_p)\ |\ \geq\ eps$ please check: Is the sign of $f\ (x_p)$ the same as the sign of $f\ (a)$. If YES, then the root is in the range $[x_p, b]$, i.e. $a\ =\ x_p$ and further calculations from point 5. If NO, then the root is inside the interval $[a, x_p]$, i.e. $b\ =\ x_p$ and further calculations from point 5.

Example: We want to approximate $\sqrt{2}$ by 4 digits after decimal point.
Since $1^2 < 2 < 2^2$ we know that $1 < \sqrt{2} < 2$ (name $a \coloneqq 1, b \coloneqq 2$)

Check whether $\sqrt{2} \in \left[1, \frac{3}{2}\right]$ or $\sqrt{2} \in \left[\frac{3}{2}, 2\right]$

Continue that process until the length of the interval is smaller than 10^{-4}. Check whether $\sqrt{2} \in \left[a, \frac{a+b}{2}\right]$ (name $a \coloneqq a, b \coloneqq \frac{a+b}{2}$) or $\sqrt{2} \in \left[\frac{a+b}{2}, b\right]$ (name $a \coloneqq \frac{a+b}{2}, b \coloneqq b$).


Start
a,b,eps loading
If f(a)f(b)<0
not
wrong numbers
a,b
yes
x_p=(a+b)/2
If |f(x_p)<eps
yes
x_p is the root
value
no
If f(x_p)f(a)>0
yes
no
a=x_p
b=x_p
Stop