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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

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Learning Plan

Topic: Percentages and Compound interest

Target Group: “Gifted” students to a high school (Y7-12).

Goal/ Content/ Description: In the industry world all companies face unexpected cases. For example, one business was doing well in recent years with strong profits, however some recent turn of events has upset this state of affairs and led to concerns around profit levels. Maybe new market or old competitors with new ideas threw a company’s whole business model up in the air overnight. So consultants often are called in to advise the actions that the company should take in order to survive and of course to maximise its profit. The consultants may advise on the potential major changes which might be required to be made, to sell the company or to sell the assets.

The goal of this learning plan is the students to be able to know how to use percentages and compound interest in order to advise a company on the correct action to be taken. The students should learn how to calculate the value of the company or the value of the assets of the company. Moreover, they will be able to find the highest expected profit for the company. In addition, it will give to students the essential skills to solve more complicated and advanced industrial problems.

Objectives:

- To develop skills to analyse the data of the real world problems.
- To develop skills to structure their approach to solve the real world problems.
- To develop skills to understand the problems.
- To develop mathematical skills to solve similar or more advanced real world problems.

Materials/ Tools:

Scientific calculators

Resources used by the teacher:

Article examples, exercises on the topic by using the Internet.

School mathematical books covering the topic

The example used in Appendix 1 found in the websites:

<https://www.myconsultingcoach.com/management-consulting-cases/case/lithium-battery-producer>.

https://sites.math.washington.edu/~nichifor/111_2012_Aut/Lecture%2024.pdf

The compound interest formula in Appendix 2 found in the website:

<http://igcseatmathematicsrealm.blogspot.com/2015/02/compound-interest.html>

Information about Net Present Values found in the website:

<https://www.investopedia.com/terms/n/npv.asp>

Resources for the student:

Article examples, exercises on the topic by using the Internet.

School textbooks covering the topic

Worksheet prepared by the teacher.

Approaches/ Methodology:

Activities Plan:

Introductory activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
Previous day	Revise the concepts of solving linear equations, percentages, ratios and compound interest.	
10 min	<p>The teacher will give the definition and the formula of the Net Present Value.</p> <p>Definition: Net present value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows over a period of time. NPV is used in capital budgeting and investment planning to analyze the profitability of a projected investment or project.</p> $\text{Net present value} = \frac{\text{Cashflow (Total Profit)}}{\text{Discount Rate}}$	<p>Students should understand the meaning of the NPV.</p> <p>Students should know that they always assume that the Discount Rate is 10% in such real world problems.</p> <p>Students should know how to calculate NPV.</p> <p>The teacher should use the given link from the Resources used by the teacher to find more information.</p>
10 min	<p>A company buys a land at \$300000 in 2015 and a year later constructs a building on it at the cost of \$200000. The land appreciates at 20% annually and building depreciates at 20% annually for the first 2 years and at 10% thereafter. The equipment of the company is worth \$25000. The company produces beers and the cost of one beer is \$0.70 and it is sold for 45% profit. The company sells 400000 beers in a year. The owner of the company, after 5 years, needs to move to another country so he needs to decide if he is to sell his company or close his company and sell his assets. What is the most profitable action that he should take?</p>	<p>Discussion on the real world problem.</p> <p>Discussion on the meaning of appreciation and depreciation.</p> <p>Discussion on the compound interest formula.</p> <p>See Appendix 1.</p>

Development activities

Time	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	The students should spend some time to structure their approach on how to solve the real world problem.	<p>Possible approach:</p> <ol style="list-style-type: none"> 1. $NPV = \frac{\text{Cashflow (Total Profit)}}{\text{Discount Rate}}$ The students should assume that the discount rate is 10%. 2. Profit = Revenues – Costs 3. Revenue = Product quantity x Price 4. Estimating Revenues 5. Estimating Profit 6. Company's assets are made up of its equipment, factory premises and the land this sits on. (That, is machinery and real estate.) 7. The students should use the compound interest equation to calculate the value of company's real estate. 8. Final Recommendation
15 min	Investigate the solution	<p>See Appendix 2.</p> <p>Discussion:</p> <p>The role of compound interest in this real world problem.</p>

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Provide a set of exercises for practise from a given handout.	<p>Discussion on problems.</p> <p>Provide solutions.</p>

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
40 min	Give a problem and ask for the solution in the classroom.	See Appendix 3. Provide solutions of the assessment.

Reflection and Closure

After this lesson, do the students feel confident and ready to solve such real world problems?

What are the advantages and disadvantages of the final recommended solution to such real problems?

Assignment for further work

Using the internet, find similar or more advanced real world problems and try to solve them and state your recommendation for the maximum profit of a company.

Appendix 1

A sequence is called GEOMETRIC (multiplicative) if the next term can be gotten from the previous one by always **MULTIPLIED by the same amount**, m called "the common ratio" (or the multiplier)

Example: 5, 10, 20, 40, ...

$$m = \frac{10}{5} = \frac{20}{10} = 2$$

$$a_1 = a_1 m^0 = 5$$

$$a_2 = a_1 m^1 = 5 \times 2$$

$$a_3 = a_1 m^2 = 5 \times 2^2$$

$$a_4 = a_1 m^3 = 5 \times 2^3$$

Then the n -th term is: $a_n = a_1 m^{n-1}$

where $n - 1$ is the number of times the common ratio is multiplied (number of steps).

Application:

If $\$P$ are invested at a rate of in **COMPOUND** interest, then the interest is applied to the entire balance. The balances then form a **geometric** sequence with common ratio $m = 1 + r$ and the balance after the interest is compounded n times is

$$S = P(1 + r)^n$$

Example:

Suppose you invest $\$800$ at an interest rate of 7%, compounded annually.

$$r = 7\% = 0.07$$

Then the common ratio is: $m = 1 + r = 1 + 0.07 = 1.07$

Your balance in year n (after $n-1$ years) is:

$$\text{Year 1: } S_1 = \$800$$

$$\text{Year 2: } S_2 = \$800 + 0.07 \times \$800 = \$800 \times 1.07 = \$856$$

Year 3:

$$S_3 = \$856 + 0.07 \times \$856 = \$856 \times 1.07 = \$800 \times 1.07 \times 1.07 = \$800 \times 1.07^3 = \$915.92$$

...etc...

More examples for geometric sequence can be found:

<https://www.geogebra.org/m/wdkmGMuJ#material/vGt8uu7A>

So in general:

COMPOUND INTEREST:

$$A = P \left(1 + \frac{r}{100} \right)^n$$

where :

A = total amount after n years

P = principal or original value

r = rate of interest per annum

n = number of years the money is invested

Example:

Peter borrows \$5000 from a bank at a rate of 9.2% per year compound interest. Calculate the amount he owes after 2 years. Give your answer correct to 2 decimal places.

Using the formula for compound interest,

$$\begin{aligned} A &= P \left(1 + \frac{r}{100} \right)^n \\ &= 5000 \left(1 + \frac{9.2}{100} \right)^2 \\ &= 5000(1.092)^2 \end{aligned}$$

$$A = 5\,962.32$$

Appendix 2

A company buys a land at \$300000 in 2015 and a year later constructs a building on it at the cost of \$200000. The land appreciates at 20% annually and building depreciates at 20% annually for the first 2 years and at 10% thereafter. The equipment of the company is worth \$25000. The company produces beers and the cost of one beer is \$0.70 and it is sold for 45% profit. The company sells 400000 beers in a year. The owner of the company, after 5 years, needs to move to another country so he needs to decide if he is to sell his company or close his company and sell his assets. What is the most profitable action that he should take?

Solution:

Estimating Revenue

$$\text{Selling price} = 0.70 \times 1.45 = \$1.015$$

$$\text{Revenues} = \text{Product quantity} \times \text{Selling price}$$

$$\text{Revenues} = 1.015 \times 400000 = \$406000$$

$$\text{Costs} = 400000 \times 0.70 = \$280000$$

Profit

$$\text{Profit} = \text{Revenues} - \text{Costs}$$

$$\text{Profit} = 406000 - 280000 = \$126000$$

Net Present Value

The students should assume that the Discount Rate is 10%.

$$\text{NPV} = \text{Cashflow} / \text{Discount Rate} = 126000 / 0.1$$

$$\text{NPV} = \$1260000$$

The company is worth \$1260000 if the owner decides to sell.

What is the value of the company's assets?

$$\text{Current value of real estate} = 300000 \times 1.2^5 + 200000 \times 0.8^2 \times 0.90^3 = \$839808$$

The value of company's machinery should then be added to establish the total value of company's assets.

$$\text{Value of company's assets} = 839808 + 25000 = \$864808$$

If company were wound up and its assets sold off, this would raise \$864808.

Final Recommendation

The owner should sell the company since the Net Present Value is higher than the value of the company's assets.

Appendix 3

NuCell is a German company producing lithium battery power units for use in electric cars made by domestic auto manufacturers.

The company has been profitable for the past few years. However, the owner of NuCell has become aware of an innovative new technology which will be entering the market within the next 18 months. This new technology promises to make equivalent power units, but storing energy as capacitors rather than lithium batteries. Such capacitor power units will offer similar or better performance for a lower production cost, due to using much cheaper raw materials.

NuCell's owner wants to know how to respond to the advent of this new technology. He is considering investing to refit NuCell's factory to produce the new capacitor units, but is also concerned that the best course of action might simply be to sell NuCell.

The following information are given:

German Electric Car Market

Cars per year manufactured Germany	6m
Ratio of conventional:electric cars produced	5:1
Average number of power units per electric car	4

Exhibit 1

NuCell has a 10% share of the market.

NuCell's Costs

Labour	10% of direct costs
Material	90% of direct costs
Cost to produce one battery unit	€200
Sale price of battery unit	€210
Fixed Costs	€2m

Exhibit 2

If NuCell invests to transition to capacitor technology, only the material costs will be reduced by 85%.

$$\text{Net Present Value} = \frac{\text{Cashflow (Total Profit)}}{\text{Discount Rate}}$$

Discount Rate is 10%.

NuCell's machinery should raise €10m if sold.

NuCell bought and set up in its current factory a decade ago

NuCell's Real Estate

Purchase cost	€2m
Value of improvements	€1m
Depreciation appreciation of property in one	5%

Exhibit 3

Solution:

The student should consider which options are open to NuCell's owner:

- Do nothing – simply continue the business as-is
- Keep operating and invest to re-fit the factory for capacitor technology
- Sell the company
- Close the company and sell its assets

The student should then set out a structured plan to decide upon which course of action to pursue. A key first step will be understanding NuCell's current position in the market.

Analysing the market and calculating the current value of NuCell

Using the data from Exhibits 1 and 2, the student can calculate the size of the German electric car battery market in terms of number of battery units sold per year.

Number of electric cars = 1/6 of national car sales = 1/6 of 6m

Number of electric cars = 1m electric cars per year

Number of batteries = 4 per electric car = 4 million

NuCell supplies 10% of these batteries.

NuCells Profits

Number of units sold by NuCell = number sold in market/market share = 4m/10

Number of units sold by NuCell = 400,000 battery units

Contribution margin = (battery price – battery cost) x number of batteries sold = (210 – 200) x 400,000

Contribution margin = €4m

Profit = contribution margin – fixed costs = 4m – 2m

Profit = 2m

Net Present Value

On the assumption of a stable market for the foreseeable future, we can calculate the Net Present Value (NPV) of NuCell as follows:

NPV = Cashflow/Discount Rate = 2m/0.1

NPV = €20m

Key Takeaway

NuCell, as a functioning business, is worth €20m if the owner decides to sell.

What is the value of NuCell's assets?

Using the data from Exhibit 3, the student can calculate the value of NuCell's real estate as follows:

Initial purchase value of NuCell real estate = €2m

Value of improvements made upon purchase = €1m

Appreciation rate = 5% per annum

Therefore, current value of real estate = $3 \times 1.05^{10} = €4.89$

The value of NuCell's machinery should then be added to establish the total value of NuCell's assets.

Value of NuCell's assets = 10m + 4.89m = €14.89m

Key Takeaway

If NuCell were wound up and its assets sold off, this would raise €14.89m

What would NuCell be worth if it invested to transition to capacitor technology?

The existing gross margin, given cost of €200 and sale price of €210 is 5%

Existing material cost per battery unit:

$$200 \times 0.9 = \text{€}180$$

$$\text{Labour cost per unit} = 200 - 180 = \text{€}20$$

$$\text{Labour cost per unit} = \text{€}20$$

New material cost per capacitor unit = old cost – 85%

$$\text{New material cost per capacitor unit} = 15\% \text{ of old cost} = 180 \times 0.15$$

$$\text{New material cost per capacitor unit} = \text{€}27$$

$$\text{New unit cost} = \text{labour} + \text{materials} = 20 + 27$$

$$\text{New unit cost} = \text{€}47$$

New sale price = new unit cost + margin

$$\text{New sale price} = 47 + (5\% \text{ of } 47)$$

$$\text{New sale price} = \text{€}49.35$$

New contribution margin = (Unit sale price – unit cost) x number sold

$$\text{New contribution margin} = 2.35 \times 400,000$$

$$\text{New contribution margin} = \text{€}940,000$$

New profit = new contribution margin – fixed costs

$$\text{New profit} = 940\text{k} - 2\text{m}$$

$$\text{New profit} = -\text{€}60,000$$

Therefore, the company now loses €60,000 per annum

Final Recommendation

The client should sell NuCell as soon as possible.

The advent of CapTech's capacitor technology will make NuCell unprofitable. The business should be sold as a going concern, as this will yield a higher value than selling off the assets.

The only alternative would be to increase sales by gaining market share. This could allow for sufficient revenues to cover fixed costs. However, this would be difficult given that the power unit market both before and after the transition will be highly commodified and cutting prices to pursue a cost advantage would only exacerbate NuCell's problems.