The following example is a guiding format for developing a Lesson Plan in a situation of supporting the students working in the context of the INNOMATH project. This mathematical content is expected to be useful for the students in their effort for solving industrial problems.

Learning Plan

Topic: Specify a particular mathematical topic

Introduction to Linear Programming: Graphical Approach

Target Group: Specify age, grade level and mathematical background of the students

"Gifted" Students in a high school at grade level 10 or 11 (4^{th} or 5^{th} year in a secondary school)

Mathematical background of the students:

Basic Concepts of Algebra,
Introductory Concepts in Coordinate Geometry,
Functions, Graphs of linear functions,
Solving linear equations and systems of equations,
Solving linear inequalities. Representing the solutions on graph diagrams
Capability of using graph software and interpreting the concepts involved
Capability of using a spreadsheet software

Goal/ Content/ Description: (Provide a brief description of the content of the lesson as well as the general goals in relation to the prospects of usefulness, applications and mathematical value of the particular mathematical topic.)

In everyday life people are interested in knowing the most efficient way of carrying out a task or achieving a goal. Many problems in real life are concerned with obtaining the best result within given constraints. In particular Advances in business and engineering research and computer technology have expanded the need of managers and industry in obtaining optimised solutions. In the business world people would like to maximize profits and minimize loss; in production, people are interested in maximizing productivity and minimizing cost. However, there are constraints like the budget, number of workers, production capacity, space, etc. Linear programming deals with this type of problems using inequalities and graphical solution method.

The goal of this lesson is to develop competencies by the students that will enable them to develop linear mathematical models, using linear equations and inequalities. By graphical approaches the students will be able to access the issue of achieving optimal solutions. Furthermore this approach will provide the framework that will enable the student to understand the whole idea of optimisation and develop the background to move into more advanced approaches, like the simplex method and so on.

Objectives: Provide a set of general mathematical objectives that can be supported by the learning process and a set of particular objectives that are the body of the topic under consideration.

General Mathematical Objectives

To develop skills for problem solving
To develop motives and positive affective tendencies for mathematics
To identify/ develop/ create applications of the related concepts and processes in the real world.
To develop mathematical skills/ through the use/ exploitation of mathematical topics or means as help/ support in modelling, calculations and representations,
To develop digital skills/ through the use/ exploitation of digital means as help/ support in calculations and representations,
To develop skills for collecting and analyzing data and other information as they appear in the real world
To exploit the flipped classroom method for supporting the various processes.

Particular Mathematical Objectives

Describe the role of mathematical models in operations decision making.
Identify and define the various concepts involved (variables, constrains, objective functions, corner points)
Describe and construct constrained optimization models.
Understand the advantages and disadvantages of using optimization models.
Describe the assumptions of linear programming.
Formulate linear programs.
Describe the geometry of linear programs.
Describe the graphical solution approach.

Materials/ Tools: for example ppt, use of graphing software etc.

Computers and or scientific calculators
Graphing software (eg Geogebra, Graph etc)
Spreadsheet
Power Point Presentations
Graph Paper

Resources used by the teacher:

Introductory books on linear programming
Articles, examples, exercises, ppt presentations, YouTube videos (eg Khan Academy) on the topic by using the Internet.
School Textbooks covering the topic
The examples in Appendix 1 and 2
Further reading and examples on the topic:

(a) FHSST Authors: The Free High School Science Texts: Textbooks for High School Students Studying the Sciences and Mathematics, Grades 10 – 12, Chapter 30
(b) P.Thie and G.E.Keough: Introduction to Linear Programming and Game Theory, WILEY (3rd edition)
(c) David Luenberger nad Yinyu Ye: Linear and Nonlinear Programming, Springer (3rd edition)

Resources for the student:

Articles, examples, exercises, ppt presentations, YouTube videos (eg Khan Academy) on the topic by using the Internet. For this the teacher is to prepare a list of webpages in the mother language of the students.
School Textbooks covering the topic.
Work sheets prepared by the teacher

Approaches/ Methodology: Describe briefly the approaches to be used (e.g. problem solving project, clipped classroom,… ,the role of the teacher, etc)

The flipped classroom approach will be used in order to give to the student the possibility for investigation, and access of information, and watching videos demonstrating the approach and posing the problem of linear programming.

The project based problem solving approach will be utilised in order to help the students to acquire the skill to investigate the prospect of extending the consideration of linear programming for non-graphical methods (in the case of more than two variables)

Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
Previous Day	Revise the concepts of solving linear equations and determining regions that are satisfied by linear inequalities	See Appendix 1 for a problem
15 min	Present a problem that is faced by the real world eg. A farmer is mixing two types of food, Type A and Type B, for his cattle. It is given that each serving is required to have 75 grams of protein and 20 grams of fat. It is known that food of type A has 15 grams of protein and 10 grams of fat and costs 60 cents per unit, and Type B contains 30 grams of protein and 5 grams of fat and costs 50 cents per unit, how much of each type should be used to minimize cost to the farmer?	Discussion on the issues Identify the important concepts **Determine:** Variables Objective functions Constrains on the variables

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Develop a plan for the solution of the problem	**What are the important processes in a linear programming approach:** Deciding on what the variables are Determining the range of values of each variable Developing the equations/ inequalities connecting them (mathematical model) Determining the Objective function/s that is/ are to be optimised (maximised, minimized or otherwise) Determining the corner points and their role

10 min	Implement the plan by using graphical methodology	
20 min	Investigate the solution Consider the problem A farmer is mixing two types of food, Type A and Type B, for his cattle. It is given that each serving is required to have 75 grams of protein and 20 grams of fat. It is known that food of type A has 15 grams of protein and 10 grams of fat and costs 80 cents per unit, and Type B contains 30 grams of protein and 5 grams of fat and costs 50 cents per unit, how much of each type should be used to minimize cost to the farmer?	Discussion The role of corner points The case of convex and concave polygons

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
40 min	Provide a set of exercises for practice from the textbook	Discussion, Solutions
10 min	Suggest the consideration of problems with more than 3 variables and suggest to work at home referring to supporting webpages (eg the Khan Academy)	

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Give a problem and ask for solution in the classroom	
	Further exercises for homework	

Reflection and Closure

What are the basic assumptions that are leading to linear programming?

What are the advantages and what are the disadvantages of using the previous approaches?

Assignment for further work

Using the Internet, Study other methods and approaches for the consideration of linear programming.

What is the Simplex Method?

Appendix 1

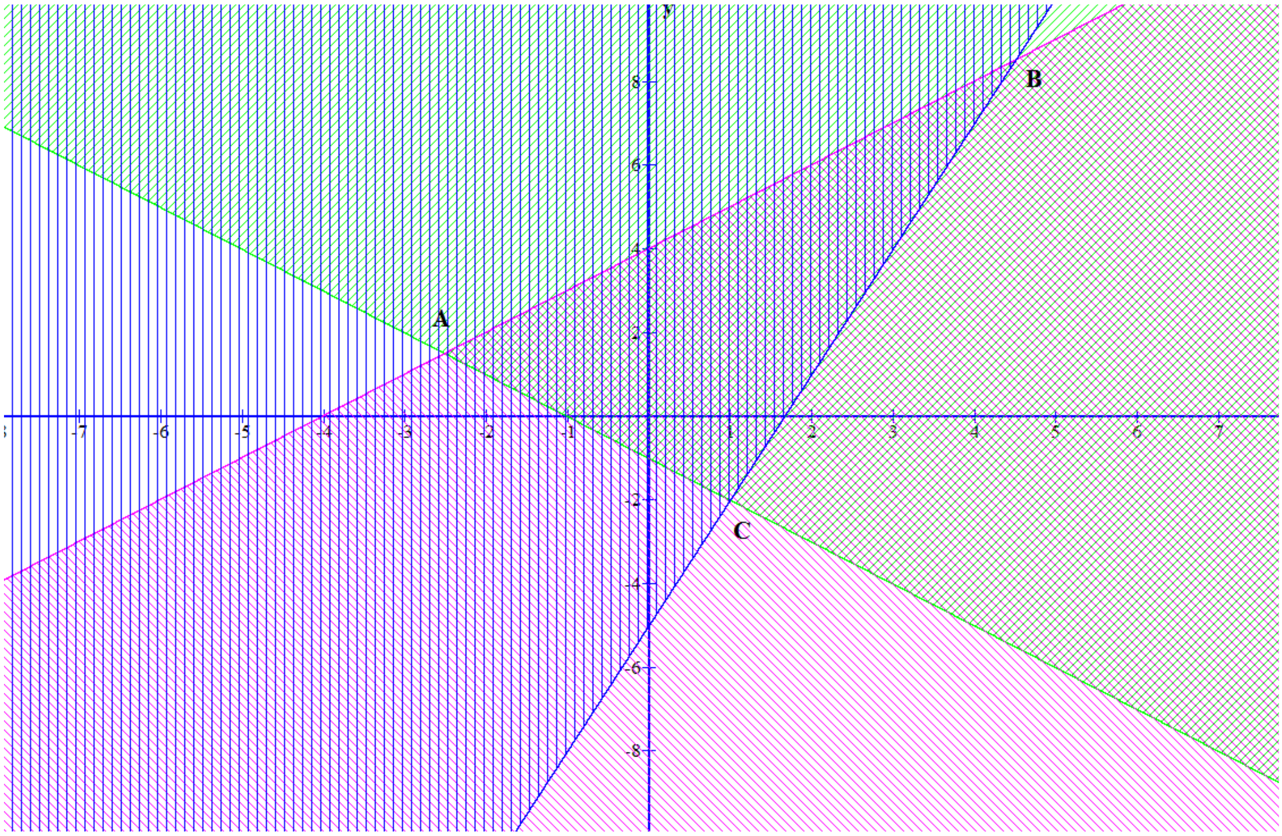
Revision

Determine the region on the plane that satisfies the constrains

y≥-x-1

y≤x+4

y≥3x-5



Constrain	Region shaded by colour	Comments
y≥-x-1	GREEN	
y≤x+4	RED	
y≥3x-5	BLUE	

Thus the region that satisfies all the constrains is the one that is on and in the interior of the triangle ABC

Appendix 2

Linear programming: The Graphical Approach

Many problems fit into the Linear Programming approach. By graphical approaches the students will be able to access the issue of achieving optimal solutions. Furthermore this approach will provide the framework that will enable the student to understand the whole idea of optimisation and develop the background to move into more advanced approaches, like the simplex method and so on.

Given a set of variables we want to assign real values to them such that they

- Satisfy a set of variable constraints represented by linear equations and/or linear inequalities
- Maximize/minimize a given linear objective function

We will consider a few examples. In these examples we want the student to become able in a) recognizing a problem which can be represented in the linear programming approach and
b) transforming it into a linear programming formulation

Problem 1

Somebody owns a chocolate shop which produces two types of box chocolates:

(a) In Normal boxes that give a €1 profit per box and
(b) In Deluxe boxes which give a €2 profit per box

There are limits in the demand per day (at most 200 normal boxes and at most 300 deluxe boxes)as well as as for the maximum production per day (at most 400 per day). The objective set is to maximise the profit.

What are the variables under consideration?

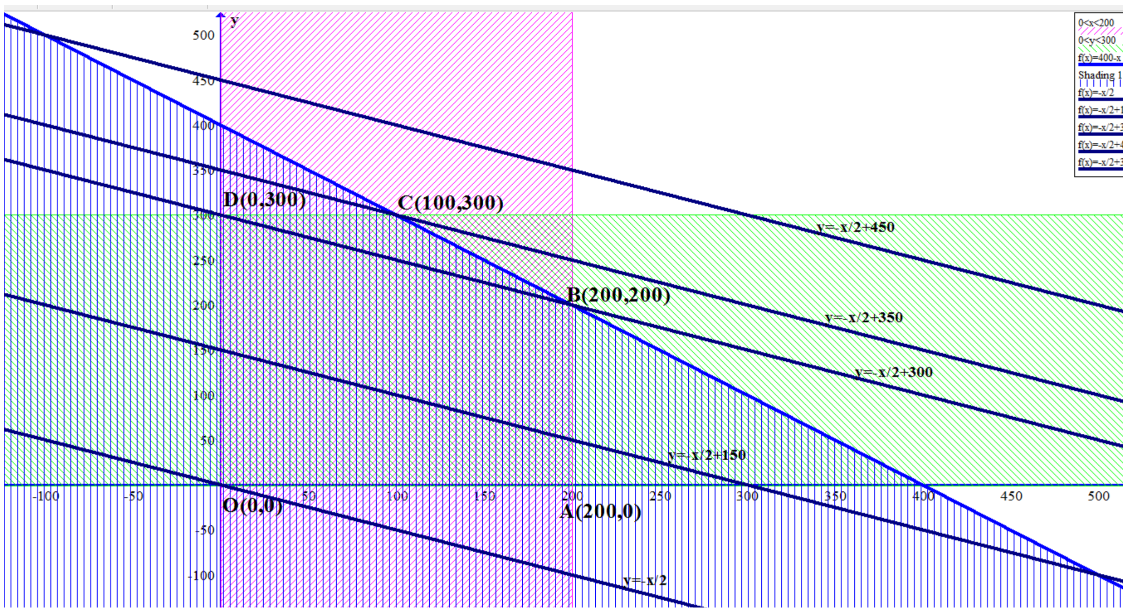
Let x be the number of normal boxes produced per day and
y be the number of deluxe boxes produced per day

What is/ are the objective function that is to be optimised (maximised or minimised)?

This is the profit which is P= x+2y
Which is targeted to become maximum

What are the constraints?

Production of normal boxes: 0 ≤ x ≤ 200 and integer
Production of deluxe boxes: 0 ≤ y ≤ 300 and integer
Maximum production capacity: x+y ≤ 400
How do represent this information graphically?



Comments:
The constraint 0 ≤ x ≤ 200 is represented by the red region on the plane
The constraint 0 ≤ y ≤ 300 is represented by the green region on the plane

The constraint $x+y \leq 400$ is represented by the blue region on the plane

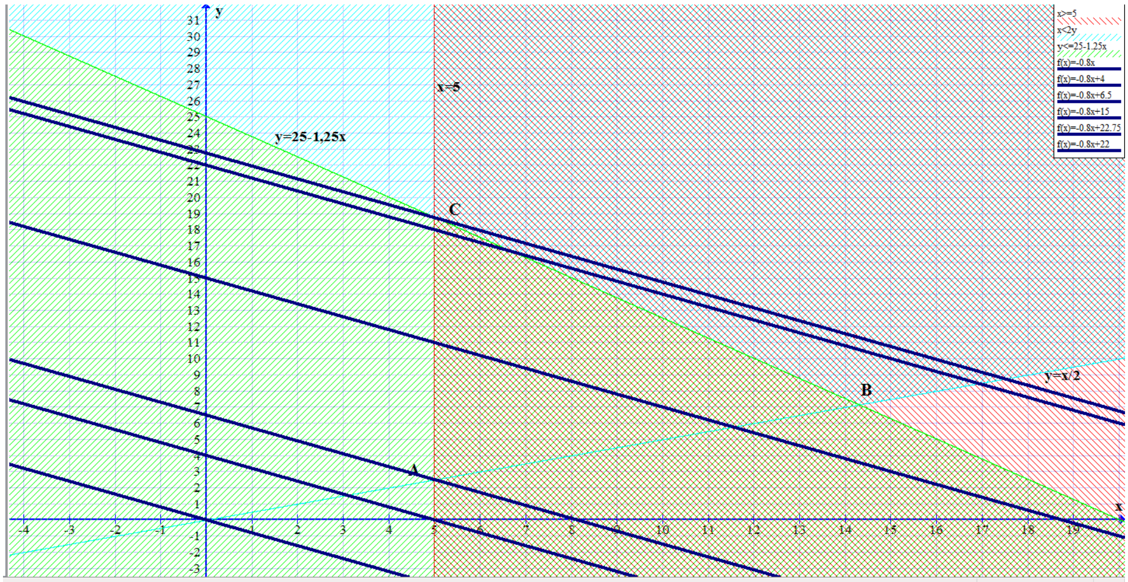
It can be seen that the constraints require that the points (x,y) (i.e the pairs of normal and deluxe boxes of chocolate) must lie in the interior or on the outskirts of the polygon OABCD with **corner points** O(0,0), A(200,0), B(200,200), C(100,300) and D(0,300).
The Profit satisfies the equation $P= x+2y$. Hence $y=-x/2+P/2$, i.e we are looking for the value of P that is maximum and the line $y=-x/2+P/2$ crosses along a region that is satisfied by all the constraints. This region in our case is defined by the polygon OABCD. In the above some of these lines are sketched. We observe that these lines are parallel with slope -1/2 (and are represented by dark blue in the present case)
,

Line $y=-x/2+P/2$	Line goes through the point	Value of P	Comments
$y=-x/2$	O	0	Profit is minimum
$y=-x/2+150$		300	The line just crosses the convex polygon
$y=-x/2+300$	B	600	
$y=-x/2+350$	C	700	
$y=-x/2+300$	D	600	
$y=-x/2+450$		900	The line does not cross the polygon

From this information we get that the maximum value of P is €700 and it occurs when x=100 and y=200.
Notes and comments
In this particular case the point C has integer values for its coordinates and thus satisfy the requirement for integers. Otherwise we should look for the line parallel to the ones with slope -1/2 that passes through a point with integer coordinates and is as near as possible to the corner C. and is inside the polygon with such values.
Hence the required optimum value of P is 700 and it occurs when x=100 and y=300

Problem 2
Mary wants to buy apples and peaches from a store. She must buy at least 5 apples and the number of apples must be less than twice the number of peaches. An orange weighs 250 gr and a peach 200 gr. Mary can carry no more than 3 kgr of fruits home. If the price of an apple is €0,4 each and that of a peach is €0,5 each, find the maximum amount of money she can spend buying the fruits
What are the variables under consideration?
Let x be the number of apples she buys
y be the number of peaches she buys
What is/ are the objective function that is to be optimised (maximised or minimised)?
The total amount she pays is $C=0{,}4x+0{,}5y$ and thus $y=2C-0{,}8x$
Which is targeted to become maximum
What are the constraints?
At least 5 apples: $x \geq 5$ and x an integer
Number of apples must be less than twice the number of peaches $x<2y$ and y an integer
Weight not more than 5 kgr: $0{,}25x + 0{,}2y \leq 5$ or $y \leq 25-1{,}25x$

How do we represent this information graphically?



Comments:
The constraint x≥5 is represented by the red region on the plane
The constraint y≥x/2 is represented by the light blue region on the plane
The constraint y ≤ 25-1,25x is represented by the green region on the plane

It can be seen that the constraints require that the points (x,y) (i.e the pairs of apple and peaches) must lie in the interior or on the outskirts of the triangle ABC with **corner points** at the intersection of the lines as following:
A: y= x/2 and x=5, B: y=x/2 and y=25-1,25x and C: x=5 and y=25 – 1,25 x .
The cost satisfies the equation C= 0,4x+0,5y. Thus y= 2C-0,8x. Hence we are looking for the value of C that is maximum and the line y= 2C-0,8x crosses along a region that is satisfied by all the constraints. This region in our case is defined by the triangle ABC. In the above some of these lines are sketched. We observe that these lines are parallel with slope -0,8 (and are represented by dark blue in the present case). As x and y are integers we are looking for such a line that passes through a grid point with integers as coordinates.

Line y=-0,8x+2C	Line goes through the point	Value of C	Comments
y=-0,8x	O the origin	0	
y=-0,8x+4	(5,0)	2	The line is below the triangle
y=-0,8x+6,5	A(5, 2,5)	3,25	
y=-0,8x+22,75	C(5, 18,75)	11,375	The line satisfies the constraint but does not pass through a grid point
y=-0,8x+22	(5, 18)	11	This point gives a maximum for C and has integers as coordinates
Y=-0,8x+21,8	(6,17)	10,9	

Hence the required optimum value of C is €11 and it occurs when x=5 and y=18.