The following example is a guiding format for developing a Lesson Plan in a situation of supporting the students working in the context of the INNOMATH project. This mathematical content is expected to be useful for the students in their effort for solving industrial problems.

Lesson/ Learning Plan

Topic: Introduction to Hypothesis testing in Statistics

Target Group: "Gifted" Students in a high school at grade level 11 or 12 (5^{th} or 6^{th} year in a secondary school)
Mathematical background of the students:
Basic concepts of Algebra,
Basic Concepts of Probability,
Basic Concepts of Statistics,
Estimation and Confidence itervals
Functions and Graphs,
Probability Distributions, in particular the Binomial, the Normal, the Student's t and the χ^2 distributions
Sampling Distributions
Capability of using a spreadsheet software and/ or Statistical packages like SPSS or Statistical tables

Goal/ Content/ Description:

During the recent pandemic on COVID-19 an issue that was the concern of many people is/ was the extent of effectiveness as well as the prospects of negative consequences of the various vaccines that were produced. It has been the object of discussion of nearly every human being. The World Health Organisation, the Governments and even simple people are/ were called to make decisions on the basis of sample information. Such decisions are called statistical decisions and form the object of inferential statistics. In attempting to reach decisions, it is useful to make assumptions or guesses about the populations involved, based on the information we get from samples. Such assumptions, which may or may not be true, are called *statistical hypotheses* and in general we are tempted to use them in the context of the probability distributions of the populations.
The goal of this lesson is to introduce the students to very simple and initial stages of this process. In this lesson we plan to consider the various concepts involved in the process of statistical hypothesis as well the various steps that have to be implemented in order to use the process. Furthermore some statistical tests for particular cases will be presented. The whole approach will give to the students the opportunity to consider and study examples that originate from issues that are of interest to the industry and the real world.

Objectives:

General Mathematical Objectives

To develop skills for problem solving
To develop motives and positive affective tendencies for mathematics
To identify/ develop/ create applications of the related concepts and processes in the real world.
To develop mathematical skills/ through the use/ exploitation of mathematical topics or means as help/ support in modelling, calculations and representations,
To develop digital skills/ through the use/ exploitation of digital means as help/ support in calculations and representations,
To develop skills for collecting and analyzing data and other information as they appear in the real world
To exploit the flipped classroom method for supporting the various processes.

Particular Mathematical Objectives

Describe the role of statistics as a means to make inferences based on evidence that is the outcome of arithmetic data
Identify and define the various concepts involved in the process of Hypothesis testing (population, sample, statistic, probability distributions and appropriate ones for particular statistics, Null and Alternative Hypotheses, confidence intervals)
Identify and explain the steps that have to be followed in the process.
Identify, explain and implement the various actions for the materialization of the steps involved in the process.
Making assumptions for a statistic and state a statistical hypothesis based on it.
Deciding on a confidence interval and on the test statistic to be used

Materials/ Tools:

Computers and or scientific calculators
Statistical Tables or Spreadsheet or Statistical Package
Power Point Presentations
Graph Paper or graphing software

Resources used by the teacher:

Introductory books on Statistics and Probability, e.g.
Spiegel and others: Probability and Statistics, Schaums Outlines Series, 4th Edition
School Textbooks covering the topic
The examples in the Appendices
Further reading and examples on the topic:

(a) Holmes and others: Introductory Business Statistics, OpenStax
(b) Rouncefield and others: Practical Statistics, Macmillan
(c) https://www.slideshare.net/darlingjunior/hypothesis-testing
(d) https://en.wikipedia.org/wiki/Statistical_hypothesis_testing
(e) https://www.tes.com/teaching-resource/hypothesis-testing-11833125

Resources for the student:

Articles, examples, exercises, ppt presentations, YouTube videos by using the Internet.
For this the teacher is to prepare a list of webpages in the mother language of the students.
School Textbooks covering the topic.
Work sheets prepared by the teacher.
The above resources for the teacher.

Approaches/ Methodology:

The flipped classroom approach will be used in order to give to the student the possibility for investigation, and access of information, and watching videos demonstrating the approach and posing the problem of testing a statistical hypothesis.

The project based problem solving approach will be utilised in order to help the students to acquire the skill to investigate the prospect of consideration test statistic in the case of industrial or finance issues or problems

Activities Plan:

Introductory activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
Previous Day	Present the issue to be considered by stating it in a form like: A major issue in statistics is how to make inferences about a population by studying and analysing samples from the population. In this process the researcher has some feeling or clue about a particular behaviour or attribute of the population. In this context it is required by him/ her to test the extent of validity of this behaviour or attribute by calculating a statistic and investigate the extent that this statistic lies in a certain interval with high expectations. Provide a set of problems faced in the real world eg: In the process of formulating their marketing strategy, a bank has some indications that first time borrowers are asking for lower loans than borrowers with a history in getting loans. So they decide to use statistics in order to test and confirm or reject the validity of these indications. A company believes that a salesperson is doing well in his/ her job if he/ she manages to promote a certain product for at least 500 euro per unit. Design a process to test the degree of fulfilling the expectations of the company by a salesperson Assign to the students to review the topic of Sampling Distributions	Ask the students to revise basic concepts of descriptive statistics and identify for the suggested and/ or other problems the following concepts/ ideas: Population Sample Assumption Statistic that can be used
20 min	Consider a problem that is faced by the real world as proposed in the previous activity eg. the first one Consider the philosophical argument and opportunity for meditation (as it is presented in Wikipedia)	Discussion on the issues Identify the important concepts involved in this activity Review the ideas of estimation and Confidence interval

	Few beans of this handful are white. Most beans in this bag are white. Therefore: Probably, these beans were taken from another bag. This is an hypothetical inference. The beans in the bag are the population. The handful are the sample. The null hypothesis is that the sample originated from the population. The criterion for rejecting the null-hypothesis is the "obvious" difference in appearance (an informal difference in the mean). The interesting result is that consideration of a real population and a real sample produced an imaginary bag. The philosopher was considering logic rather than probability. To be a real statistical hypothesis test, this example requires the formalities of a probability calculation and a comparison of that probability to a standard	Discussion and reflection on the ideas and concepts that form the basis of the topic

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
15 min	Present a Definition of a hypothesis It is a statement/ assumption about one or more populations. It is usually concerned with the parameters of a population and it is stated in such a way that it provides the forum for evaluation of its truthfulness or not by appropriate statistical techniques. There are two hypotheses involved in hypothesis testing **1. the Null hypothesis H_0**: It is the hypothesis to be tested . **2. the Alternative hypothesis** H_1: It is a statement of what we believe to be true if our sample data cause us to reject the null hypothesis	Discussion on the important concepts involved i.e. population -parameter, sample-statistic, Ask the students to identify parameters and statistics and propose these two statements for the problems expressed earlier
10 min	Illustrate the process by referring to the following issue **Members of a jury have to decide whether a person is guilty or innocent based on evidence** **Null hypothesis:** The person is innocent **Alternative hypothesis:** The person is not innocent (i.e. guilty) Can we be sure that the verdict (decision) is right?	Observe that The null can only be rejected if there is enough evidence to doubt it i.e. the jury can only convict if there is beyond reasonable doubt for the null of innocence

<table>
<tr><td></td><td></td><td>They do not know whether the person is really guilty or innocent so they may make a mistake</td></tr>
<tr><td>deci</td><td>In view of the previous discussion it becomes obvious that there is always a probability of error in the process of hypothesis testing. Actually in the road towards reaching a decision we are in a cloud that can be described by the following table:
<table>
<tr><th></th><th>H_0 is accepted/ adopted</th><th>H_0 is rejected</th></tr>
<tr><td>H_0 is true</td><td></td><td></td></tr>
<tr><td>H_A is true</td><td></td><td></td></tr>
</table>
Then based on the data we can reach a decision described in the following
<table>
<tr><th></th><th>H_0 is accepted/ adopted</th><th>H_0 is rejected</th></tr>
<tr><td>H_0 is true</td><td>Correct decision</td><td>Type II ERROR</td></tr>
<tr><td>H_A is true</td><td>TYPE I ERROR</td><td>Correct decision</td></tr>
</table>
The probabilities of the decision for these situations are
<table>
<tr><th></th><th>H_0 is accepted/ adopted</th><th>H_0 is rejected</th></tr>
<tr><td>H_0 is true</td><td>P(H_0 is accepted/ H_0 is true)</td><td>P(H_0 is rejected/ H_0 is true) = level of significance</td></tr>
<tr><td>H_A is true</td><td>P(H_0 is accepted/ H_A is true)=1-p</td><td>P(H_0 is rejected/ H_A is true)=level of significance of the test=p</td></tr>
</table></td><td>• What is the level of significance?
• What is a Type I error? A Type II error?
• What is a false positive?
• What is statistical power?</td></tr>
<tr><td>20 min</td><td>What are the important stages for hypothesis testing?
Step 1: Define the problem
Step 2: State/ Decide H_0 and H_A
Step 3: Calculate a test statistic
Step 4: Calculate a p value
Step 5: Make a decision and interpret your conclusions</td><td>Discussion and stating the various stages
Implement these in the case of the problems set at the starting point</td></tr>
</table>

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
40 min	Provide a set of exercises for practice from the textbook	Discussion, Solutions

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
10 min	Give a problem and ask for solution in the classroom	
	Further exercises for homework	

Reflection and Closure

What are the basic assumptions that are leading to hypothesis testing?

What are the advantages and what are the disadvantages of using the previous approaches?

Assignment for further work

Using a statistical package (eg SPSS) Find solutions to the above problems