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INNOMATH: Innovative enriching education processes for Mathematically Gifted Students in Europe

Reference number: 2019-1-DE03-KA201- 059604

Implementation period: **September 2019 – August 2021**

Training program for teachers or others interested in the identification/ support/ enriching with practical skills of mathematically gifted students in the context of the INNOMATH project

Template

Module Number and Area/ Topic: Module 2

Inquiry Based Learning

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

This module focuses on the approach of inquiry learning. Students are invited to create facts and data, to organise it and infer from it generalization, rules and principles. It stems from preliminary data collection to induce the process. The need for data organization is primordial and should not be dropped from a metaphorical helicopter, because most of the used formulae are supposed to be known but they are actually not available to be used. Teachers, as guide on the side rather than sage on the stage, guide the group of learners to form relevant questions and to shape conjectures using critical thinking. Helping students voice their thoughts, hear one another and appreciate each other's ideas is important in the process.

Not spoiling the activity is a very delicate balance therefore the worksheet is to be first well understood by the teacher and be co-constructed with students rather than used as is.

This approach can be used at basic level to deeply root important concepts that otherwise *fall from the sky* as a *deus ex machina* that actually don't make much sense. This is why we present as examples, at a basic level, the context of constructing the meaning of the formula of the volume being linearly dependent on

any independent side lengths or rather, the *cube* of the size of an object. At a more advanced level, we investigate continued fractions of square roots of integers through origami. But both modules culminate in the same final activity: the discovery of the volume of a regular tetrahedron.

Derivation of volume growth with respect to enlargement

Students are guided to discover that, depending on the dimension, enlarging an object has a non linear effect on its measure (area or volume). Constructing paper models of different objects of different sizes, students gather data and arrive at the conclusion that the enlargement factor is used multiplicatively three times for the volume. Their findings is put to the use for the discovery of a formula for the volume of a regular tetrahedron, through an origami 3D puzzle.

Learning Outcomes: With the completion of this module the trainees should be able to
1. understand that, under enlargement, the volume of an object depends on the cube of the enlargement factor;
2. construct paper origami models of regular tetrahedra and octahedra;
3. find a formula for the volume of a regular tetrahedron.

Content and Resources:

1. Simple volume growth
    - worksheet page 1
    - nets of simple volumes.
2. Volume growth of any shape
    - worksheet page 2
3. The volume of the tetrahedron
    - worksheet page 3
    - geogebra book

Methodology and approaches for the module training presentation:

The methodology of the module is based on the approach of inquiry learning as a process of active construction of knowledge.
The presentation gives an overview of the topic, target group, content-related prerequisite, step sequence of the module and its objectives, materials and respective processing time.

Instruments/ Tools/ Supporting Material/ Resources to be used:

- worksheet (each of the three steps comprises one page)
- worksheet (suggested solution)
- geogebra book
- sheets of paper
- a bucket of sand, semolina, lentils, chick peas or any other flowing material
- plastic balloons, water bucket, camera

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 0 Introductory	
Development	In the introduction, ideas around the notions of physical quantities such as length, area, volume are gathered, together with their units and meaning of cm, cm^2, cm^3. Formula for the volume of a cuboid is retrieved, especially its linear dependency with regard to each dimension: 1x2, 2x2, 2x2x2.
Materials	12 squares, 8 or better 27 dice.
Resources	Presentation (ppt)
Estimated Time	5 min
Environment/Room Setting	frontal space/ general introduction/ demonstration
Trainees' role	activate previous knowledge, express oneself, gather ideas

Development activities with integrated practical activities (hands-on activity)

Activity Number and broad Description: 1 Simple volume growth	
Development	Students constructs different simple volumes (cuboid, cone, pyramid, cylinder) at a given unit scale and its double. They count how many times the smaller one fills the larger one with sand (or semolina). They conjecture the multiplication by a factor 8 of the volume when doubling the size. For fast students: an enlargement factor 3/2 or another one.
Materials	- worksheet page 1 - strong paper (cardboard), scissors and tape, sand or semolina, buckets.
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Activity Number and broad Description: 2 Volume growth of any shape	
Development	Students try out their hypothesis on other shapes, especially the sphere. A graph of its volume with respect to its size can summarize the data.
Materials	- worksheet page 2 - digital camera (smartphone, webcam) or projection screen - inflatable balloons, water, buckets, measuring jug
Resources	Presentation (ppt)
Estimated Time	approx. 30 min
Environment/Room Setting	own workspace/ individual work phase, need of a sink, a basinet or outside
Trainees' role	active role active construction of knowledge and skills

Evaluation of Learning Outcomes

Activity Number and broad Description: 3 The volume of the tetrahedron	
Development	Students construct an origami model of tetrahedra and octahedra. Solving a little 3D puzzle for doubling the tetrahedron with these pieces, they infer that a certain cube, of size $\sqrt{2}$ a, contains 24 tetrahedra of size a. This allows to infer that the volume of the tetrahedron is $(\sqrt{2}\ a)^3/24$.
Materials	- worksheet page 3, tetrahedron and octahedron folding instructions - geogebra book - paper
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ collective work phase
Trainees' role	active role active construction of knowledge and skills

Reflection and Closure activity:

In order to make the learning success of this module visible, the students should share their individual insights gained through this learning module in their own words in a padlet. This collaborative method is best suited because all participants of this module can access it at the same time and read and discuss each other's contributions. In doing so, the padlet offers the possibility to prepare these individual reflections in written form as text, auditorily as a voice message, visually as a graphic or audio-visually as a video. These contributions can also be networked so that connections become clear. The teacher can take up these presented contributions in a joint final discussion and correct or deepen them if necessary.

Source:

IREM de Lyon

Continued fraction of square roots of integers

Students are guided to discover that looking at a positive real number as the ratio of a rectangle allows to experimentally get a rational approximation for it. They investigate the A4 paper format and discover that its ratio is $\sqrt{2}$ and, with the help of folding paper, compute its continued fraction approximation. Its diagonal is in a ratio of $\sqrt{3}$. Likewise, we construct a paper of ratio $\sqrt{3}$ and infer its continued fraction expansion. Understanding geometrically square roots of integers, we set the scene for their continued fraction expansion computations. Their findings is put to the use for the discovery of a formula for the volume of a regular tetrahedron, through an origami 3D puzzle.

Learning Outcomes: With the completion of this module the trainees should be able to

1. understand that the A4 paper is of ratio $\sqrt{2}$ and why;
2. compute the continued fraction expansions of $\sqrt{2}$, $\sqrt{3}$ and maybe $\sqrt{n}$;
3. construct paper origami models of regular tetrahedra and octahedra;
4. find a formula for the volume of a regular tetrahedron.

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