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INNOMATH: Innovative enriching education processes for Mathematically Gifted Students in Europe

Reference number: 2019-1-DE03-KA201- 059604

Implementation period: **September 2019 – August 2021**

Training program for teachers or others interested in the identification/ support/ enriching with practical skills of mathematically gifted students in the context of the INNOMATH project

Template

Module Number and Area/ Topic: Module 3

Discovery Based Learning

Derivation and modeling of constrained growth processes with the natural exponential function

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

This module focuses on the approach of (self-)discovery learning. In contrast to teacher-centred, rather passive learning, the focus is on the students' own activities. After a short description of the tasks, the students should solve them independently and work with the materials provided. The teacher deliberately takes a step back and gives room to the momentum of the solution process. Through this, networked knowledge can emerge through the active construction of knowledge. This approach pursues the goal that learning is a subjective, active-discovering and constructive process.

The discovery of special exponential functions is used as an exemplary context. Starting from known exponential functions, the students analyse relations between the functions and their derivative with Geogebra. They make assumptions about a special exponential function and finally derive Euler's number e using the differential quotient. Then the students repeat (known) simple derivation rules in the new context of exponential functions and learn the chain rule. They use these to form

the derivative of functions at different levels of difficulty. Finally, students analyse graphical representations and derive the equations for constrained and logistic growth functions. They work through several exercises (also in factual relations) and give function equations, determine growth rates and learn about properties of growth concepts.

Learning Outcomes: With the completion of this module the trainees will be able to

1. derive exponential functions qualitatively, name Euler's number e as a special basis of an exponential function and derive it as a limit value of a sequence.
2. derive exponential functions using various derivation rules.
3. describe equations for constrained and logistic growth functions and explain their properties (in factual contexts) also computationally.

Content and Resources:

Derivation und modelling of constrained growth processes with the natural exponential function

1. Derivation of the (natural) exponential function
    - worksheet page 1
    - geogebra file 1
    - youtube videos
      Derivation and summary of Euler's number e:
      https://www.youtube.com/watch?v=m2MIpDrF7Es
      Excursus on further properties and connections of Euler's number e:
      https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
2. Derivation rules for exponential functions
    - worksheet page 2
    - youtube video
      Proof of the chain rule:
      https://www.youtube.com/watch?v=m0LZX19DyyI
3. Modelling constrained and logistic growth
    - worksheet page 3
    - geogebra file 2
    - calculator
    - if available, but not necessary: computer algebra system (CAS)

Methodology and approaches for the module training presentation:

The methodology of the module is based on the approach of self-discovery learning as a process of active construction of knowledge.
The presentation gives an overview of the topic, target group, content-related prerequisite, step sequence of the module and its objectives, materials and respective processing time.

Instruments/ Tools/ Supporting Material/ Resources to be used:

- worksheet (each of the three steps comprises one page)
- worksheet (suggested solution)
- geogebra file 1
- geogebra file 2
- video: Derivation and summary of Euler's number e: https://www.youtube.com/watch?v=m2MIpDrF7Es
- video: Excursus on further properties and connections of Euler's number e: https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
- video: Proof of the chain rule: https://www.youtube.com/watch?v=m0LZX19DyyI
- presentation (PPT)
- padlet: https://kant-gymnasium.padlet.org/okaufmann/jmjw07stw5mli2uy

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 0 Introductory	
Development	The introduction is done through a short demonstration of exponential functions and their derivatives using geogebra as well as visual impressions of key words and associated graphics. In doing so, the students should generate a reality reference as well as motivating internal mathematical questions and activate their previous knowledge of exponential functions and the concept of derivatives.
Materials	Geogebra file 1, graphics, keywords
Resources	Presentation (ppt)
Estimated Time	5 min
Environment/Room Setting	frontal space/ general introduction/ demonstration
Trainees' role	passive role activate previous knowledge

Development activities with integrated practical activities (hands-on activity)

Activity Number and broad Description: 1 Derivation of the (natural) exponential function	
Development	The students discover the relationship between known exponential functions and their derivatives using Geogebra. They make assumptions about a special exponential function and finally derive Euler's number e using the differential quotient.
Materials	- worksheet page 1 - geogebra file 1 - youtube videos *Derivation and summary of Euler's number e:* https://www.youtube.com/watch?v=m2MIpDrF7Es *Excursus on further properties and connections of Euler's number e:* https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
Resources	Presentation (ppt)

Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Activity Number and broad Description: 2 Derivation rules for exponential functions	
Development	The Students repeat (known) simple derivation rules in the new context of exponential functions and learn the chain rule. They use these to form the derivative of functions of varying degrees of difficulty.
Materials	- worksheet page 2 - youtube video *Proof of the chain rule:* https://www.youtube.com/watch?v=m0LZX19DyyI
Resources	Presentation (ppt)
Estimated Time	approx. 30 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Evaluation of Learning Outcomes

Activity Number and broad Description: 3 Modelling constrained and logistic growth	
Development	The Students analyse graphical representations and derive the equations for constrained and logistic growth functions. They work on several exercises (also in factual contexts) and give function equations, determine growth rates and learn about properties of the growth concepts.
Materials	- worksheet page 3 - geogebra file 2 - calculator - if available, but not necessary: computer algebra system (CAS)
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Reflection and Closure activity:

In order to make the learning success of this module visible, the students should share their individual insights gained through this learning module in their own words in a padlet. This collaborative method is best suited because all participants of this module can access it at the same time and read and discuss each other's contributions. In doing so, the padlet offers the possibility to prepare these individual reflections in written form as text, auditorily as a voice message, visually as a graphic or audio-visually as a video. These contributions can also be networked so that connections become clear. The teacher can take up these presented

contributions in a joint final discussion and correct or deepen them if necessary. Here you can find the link to the prepared padlet:

https://kant-gymnasium.padlet.org/okaufmann/jmjw07stw5mli2uy

Sources:

- KIRA Deutsches Zentrum für Lehrerbildung Mathematik (2021): *Entdeckendes Lernen im Mathematikunterricht.* In: https://kira.dzlm.de/lernen-wie-kinder-denken/entdeckendes-lernen-im-mathematikunterricht
- Brandt, D. et al. (2016): *Lambacher Schweizer Mathematik. Oberstufe mit CAS-Einsatz*. 1st edition. Ernst Klett Verlag. Stuttgart, p. 224 – 233.