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INNOMATH: Innovative enriching education processes for Mathematically Gifted Students in Europe

Reference number: 2019-1-DE03-KA201- 059604

Implementation period: **September 2019 – August 2021**

Training program for teachers or others interested in the identification/ support/ enriching with practical skills of mathematically gifted students in the context of the INNOMATH project

Template (*for the Methodology and Structure of a Learning Plan for Presenting a Module for the INNOMATH course programme, module of 3 hours duration).*

Module Number and Area/ Topic:

M10- Cooperative Learning for teachers and students

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

Collaborative learning is a method of teaching and learning in which students work together, sometimes in pairs, sometimes in small groups, to solve one and the same problem, to explore a new topic or launch new ideas, new combinations or even innovations. authentic.

Cooperative learning is specific collaborative learning:

- students work in small groups;
- the activity is structured;
- students are evaluated for individual work;
- the work done by the whole group is also appreciated;
- students communicate directly with each other - face to face;
- students learn to work as a team.

In small groups, students can arm themselves and develop their skills, learn to communicate effectively, solve problems. When cooperation groups are guided by

clear objectives, students engage in many types of activities, which contributes to a better mastery of the topics explored.
In order to create an appropriate environment for cooperative learning, three conditions must be met:
1. students need to feel confident but challenged;
2. groups must be small in order for everyone to be able to contribute;
3. The task of the group must be clear.

Learning Outcomes: With the completion of this module the trainees will be able to:

1. Superior performance and increased retention capacity.
2. More frequent higher order reasoning, deeper understanding and critical thinking.
3. Better focus on learning and reducing disruptive behavior.
4. Increased motivation for performance and intrinsic motivation for learning.
5. Increased ability to see one situation from another's perspective.
6. Better, more tolerant relationships with colleagues, regardless of ethnicity, gender, intellectual abilities, social class or condition.
7. State of psychological comfort, ability to adapt.
8. Self-confidence, based on self-acceptance.
9. High social skills.
10. Positive attitude towards study subjects, studies and school.
11. Positive attitude towards teachers, managers and other people in the school.

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration):

- **Collaborative learning guide for teachers and students**

Methodology and approaches for the module training presentation:

- **Collaborative learning: group rules, badge method, brainstorming, I know / I want to know / I learned / the gallery tour, debates, group work**
- **Constructionism: research-based and project-based learning**

Instruments/ Tools/ Supporting Material/ Resources to be used:

(list of file, web links, videos, **M10- Cooperative Learning for teachers and students**

What is collaborative learning | Collaborative learning strategies || SimplyInfo.net

https://www.youtube.com/watch?v=7gWS4mxM1Qc

Collaborative Learning Vs Co-operative Learning (English)

https://www.youtube.com/watch?v=QgxfAh1S38g

Co-operative Learning - Origin and Concept

https://www.youtube.com/watch?v=7zKNpqGlhd0

Pedagogical/Learning Sequencing and Activities Plan: ……..

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: Activity 1: BASIC ELEMENTS OF COLLABORATIVE LEARNING	
Development	At the beginning of the working session, the trainers establish the specific rules of communication with the participants - Each participant receives a post that will convey their name, organization of origin, country and a word to define collaborative learning between students and teachers. Participants justify their choice. - Theoretical session: trainers will present the approach to cooperative learning (the presentation starts with a film about the origin of cooperative learning). - Discussion and reflection: the trainers will stimulate the discussions between the participants through reflection questions. - Brainstorming participants complete the first two columns of the I know / want to know about collaborative learning method, and the last column will be completed at the end of the work session after participants provide feedback on what they have learned
Materials	Slides, Videos, posters, flipchart sheets
Resources	Videoprojector, Internet connection, sheets, pens
Estimated Time	30 minutes
Environment/Room Setting	plenary - individual
Trainees' role	The trainers will present the contents and stimulate participants' active reflection and debate on the addressed topics.

Development activities

Activity Number and broad Description: Activity 2 THE ROLE OF STUDENTS in collaborative learning	
Development	Within each group, the roles played by the students can be oriented towards solving the task, maintaining the group or both. Because students need to learn behaviors necessary to perform both roles, sometimes the teacher explicitly distributes the roles in the group, for example: CONTROLLER: Make sure the group members understand what it works. RESEARCHER: Look for the necessary information in other groups or, sometimes even to the teacher. CHRONOMETER: Make sure the group focuses on the task and comply with the time limits fi xate. hearer ACTIVE: Repeat or rephrase what the other members of the group have said. INVESTIGATOR: Ask questions to get ideas or task-related comments from all group members. SUMMARY: Order the group's conclusions in such a way that all in all they make sense. ENCOURAGER: Congratulates, helps and encourages all members group.

	OFFICER OF MATERIALS: Divide and collect group materials. READER: Read any written material to the group. Other roles can be invented, depending on the species of the activity performed. For example, there may be a REPORTER, whose job it is to communicate the group's findings to the whole class; an OPPONENT, who must suggest alternatives to the solution proposed by the group, etc.
Materials	Slides, Videos, posters, flipchart sheets
Resources	Videoprojector, Internet connection, sheets, pens
Estimated Time	30 minutes
Environment/Room Setting	plenary - individual
Trainees' role	The trainers will present the contents and stimulate participants' active reflection and debate on the addressed topics.

(add more Activity sections as needed)

Practicing Activities (hands-on activity)

Activity Number and broad Description: Activity 3 Motivation techniques and strategies for students actively involved in the learning process in learning mathematics	
Development	Motivational teachers to develop students' understanding of fundamental concepts - not memorizing facts, definitions and procedures - as a first priority. Many of the classroom strategies used by these teachers have proven to be both cognitive science and learning research the best methods to help students build and use knowledge in math and science. This report describes five of these strategies, called contextual teaching strategies: • **Relationships** - learning in the context of life experiences or pre-existing knowledge • **Experimentation** - learning by doing or through exploration, discovery and invention • **Application** - learning by using concepts • **Cooperation -** learning in the context of sharing, responding and communicating with other learners • **Transfer** - the use of knowledge in a new context or in a new situation - one that has not been covered in class
Materials	Slides, Videos, posters, flipchart sheets Collaborative Learning Vs Co-operative Learning (English) https://www.youtube.com/watch?v=QgxfAh1S38g
Resources	Videoprojector, Internet connection, sheets, pens
Estimated Time	30 minutes
Environment/Room Setting	plenary - individual
Trainees' role	The trainers will present the contents and stimulate participants' active reflection and debate on the addressed topics.

Evaluation of Learning Outcomes

Activity Number and broad Description: Activity 4 Examples of good practice - Techniques and motivational strategies for students actively involved in the process of learning mathematics	
Development	**RELATING**

Relating is the most powerful contextual teaching strategy. It is also at the heart of constructivism. Relating is learning in the context of one's life experiences or preexisting knowledge.

Excellent teachers plan carefully for learning situations in which students can experience felt meaning. Careful planning is needed because often students do not automatically connect new information to the familiar. Research shows that, although students may bring memories or prior knowledge that is relevant to a new learning situation, they can fail to recognize its relevance.8

When teachers both provide environments in which students activate memories or prior knowledge and recognize the relevance of the memories or knowledge, they are using relating.

As an example, consider a mathematics lesson on ratio and proportion. A traditional approach typically begins with a definition, followed by an example:

A teacher using relating could begin the lesson by asking questions that almost every student can answer from life experiences outside the classroom: "Have you ever made fruit punch from frozen concentrate? What did the instructions say?" The teacher then reinforces the students' prior knowledge by reading the instructions from a real container.

When a teacher relates this familiar experience to the definition of ratio, students can immediately see the relevance of their prior knowledge. Most students feel that they already know about ratio, or that the concept of ratio is accessible, because they are familiar with the experience of making fruit punch. They are also more likely to remember the definition of ratio because they can relate it to the fruit punch instructions.

EXPERIENCING

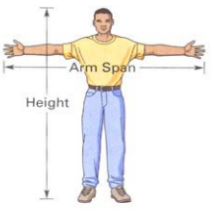
Relating connects new information to life experiences or prior knowledge that students bring with them to the classroom. But this approach is not possible if students do not have relevant experience or prior knowledge. Teachers can overcome this obstacle and help students construct new knowledge with orchestrated, hands-on experiences that take place inside the classroom. This strategy is called experiencing. It is learning by doing— through exploration, discovery, and invention. In-class hands-on experiences can include the use of manipulatives, problem-solving activities, and laboratories. Manipulatives. These are simple objects that students can move around to model abstract concepts concretely. For example, in mathematics, base-ten blocks model numeric representation in the decimal system. Fraction bars demonstrate the meaning of simple fractions and addition and multiplication of fractions. Area tiles model multiplication of Page 6 polynomials. Some computer programs, such as Geometer's Sketchpad and Cabri, can be considered manipulatives since they enable students to visualize and explore concepts and to quickly see answers to "what if ? " questions. Manipulatives have been shown to enhance student performance when they are coherently integrated into the curriculum.

Problem-solving activities. These are learning experiences that engage students' creativity while they are learning key concepts. These

activities also teach problemsolving skills, analytical thinking, communication, and group interaction. The best problem-solving activities introduce key concepts—usually curriculum objectives or standards—as they arise naturally in problem situations. This allows students to see a need or a reason for using the new concepts. When they see relevant uses of knowledge in solving interesting problems, students can make sense of what they are learning.18 This has been shown to motivate students to exert the required effort to gain and use the new knowledge.19 An example of this kind of problem-solving activity is a continuation of the lesson on ratio and proportion. After introducing the concept of ratio using relating and the instructions for making fruit punch, the teacher poses a problem: "How many cans of concentrate and how many cans of water are needed to make fruit punch for the whole class?" Students working individually or in groups are likely to try several different problem-solving approaches and have different solutions, because answers depend on students' assumptions. (How much punch is needed? How can we make sure we use the same 3 : 1 ratio of water to concentrate?) Relevance and motivation can be enhanced if students know that at the end of the lesson they will, as a class, decide on a single best solution and then actually make the fruit punch to "check their answer." This activity can create motivation for the need to know about and use ratio and proportion, but students are very unlikely to discover the key mathematical concepts on their own. The teacher must be prepared to facilitate student discussion and problem solving, summarize students' approaches and results, and demonstrate and generalize the concept at the right time. In mathematics, definitions and solution procedures are part of this generalization. Generalizing the specific experience or information is a key step in learning. Research shows that students have a greater ability to use new knowledge in multiple contexts, beyond those covered in class, when the teacher (or, if possible, the Page 7 student) generalizes key information after the student uses the information or experience in a specific context.20 The ability to use new knowledge in novel situations is called transferring, and we will discuss it more later as a separate contextual teaching and learning strategy.

Laboratory activities and projects. These are usually longer and require more planning than problem-solving activities. In a laboratory, students work in small groups to collect data by making measurements, analyze the data, make conclusions and predictions, and reflect on the fundamental concepts involved in the activity.

Students can be involved in laboratory activities even in mathematics classes. For example, in one classic activity students in groups measure their heights and arm spans. They combine their group data with the rest of the class and display the class data in a chart. A chart is one way to represent the data. The students then make a coordinate axis system and plot the (height, arm span) ordered pairs.



APPLYING

We define the applying strategy as learning by putting the concepts to use. Obviously, students apply concepts when they are engaged in hands-on problem-solving activities and projects like those described above. Teachers also can motivate a need for understanding the concepts by assigning realistic and relevant

exercises. These exercises are “word problems” like those found in all textbooks. But they have two major differences: They pose very realistic situations, and they demonstrate the utility of academic concepts in some area of a person’s life. Both are important for application problems to be motivational. For example, the following is a typical word problem a geometry lesson on volume of solids. It may be “real world,” but, after assigning this problem, how would you answer a student asking, “So what?” A hemispherical plastic dome covers an indoor swimming pool. If the diameter of the dome measures 150 feet, find the volume enclosed by the dome in cubic yards. The intent of the problem is to have students recall and use the formula for the equation of the volume of a sphere. The problem statement below also requires students to recall and use this formula. But in this problem, the formula and calculation are crucial in a believable decision-making situation. This problem inherently answers, “So what?” Montgomery is a pharmacist at a pharmaceutical manufacturing plant. He is responsible for selecting the correct capsule sizes for the company’s products. The capsule size determines the dosage. The company uses eight sizes. The body length lB, cap length lC, and diameter d of the capsules are shown in the table.

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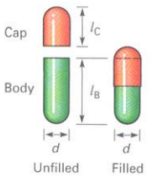

Capsule Size	Body Length (mm)	Cap Length (mm)	Diameter (mm)
000	22.96	13.44	9.52
00	20.50	12.00	8.50
0	18.86	11.04	7.82
1	16.51	9.65	6.86
2	15.35	9.10	6.25
3	13.60	8.13	5.47
4	12.30	7.20	5.10
5	9.84	5.76	4.08

Research shows that realistic or authentic exercises can motivate students to learn academic concepts at a deeper level of understanding. Recommended classroom strategies from research include: • “Focus on meaningful aspects of learning activities. Teachers should stress how the academic tasks that are done in the classroom are relevant and ‘authentic’ tasks that have meaning in the real world.”29 • “Design tasks for novelty, variety, diversity, and interest. Teachers should attempt to provide a wide variety of tasks for students to engage in and ensure that the tasks have some novel, interesting, or surprising features that will engage the students.”30 • “Design tasks that are challenging but reasonable in terms of students’ capabilities.”

COOPERATING

Many problem-solving exercises, especially when they involve realistic situations, are complex. Students working individually sometimes cannot make significant progress in a class period on

	these problems. They can become frustrated unless the teacher provides step-by-step guidance. On the other hand, students working in small groups can often handle these complex problems with little outside help.35 Teachers using student-led groups to complete exercises or hands-on activities are using the strategy of cooperating—learning in the context of sharing, responding, and communicating with other learners. Working with their peers in small groups, most students feel less self-conscious and can ask questions without feeling embarrassed. They also will more readily explain their understanding of concepts to others or recommend problem-solving approaches for the group. By listening to others in the group, students reevaluate and reformulate their own sense of understanding. They learn to value the opinions of others because sometimes a different strategy proves to be a better approach to the problem. When a group succeeds in reaching a common goal, student members of the group experience higher selfconfidence and motivation than when students work alone. **TRANSFERRING** In a traditional classroom, the teacher's primary role is to convey facts and procedures. The students' roles are to memorize the facts and practice the procedures by working skill drill exercises and, sometimes, word problems. Students who can recall and repeat the appropriate facts and procedures score well on the end-of-unit or end-of-semester test. By contrast, in a constructivist or contextual classroom, the teacher's role is expanded to include creating a variety of learning experiences with a focus on understanding rather than memorization. Contextual teachers use the strategies discussed above (relating, experiencing, applying, and cooperating) and they assign a wide variety of tasks to facilitate learning for understanding. In addition to skill drill and word problems, they assign experiential, hands-on activities and realistic problems through which students gain initial understanding and deepen their understanding of concepts. Students who learn with understanding can also learn to transfer knowledge. Transferring is a teaching strategy that we define as using knowledge in a new context or novel situation—one that has not been covered in class.
Materials	Videoprojector, Internet connection, sheets, pens
Resources	worksheets on each strategy
Estimate Time	60 minutes
Environment/Room Setting	plenary – individual and work groups
Trainees' role	The trainers will present the contents and stimulate participants' active reflection and debate on the addressed topics.

Reflection and Closure activity: