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INNOMATH

Innovative enriching education processes for Mathematically Gifted Students in Europe

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Learning Plan

Topic: Spherical Geometry

Target Group: Students at Grade 10 to 12 (13) (age range: 15-18 years old)/ Form 4 to 6, in a secondary school

Goal/ Content/ Description: In order to help the students comprehend some differences between flat Euclidean geometry and spherical geometry and learn a way of computing distances and triangular surfaces on a sphere from given spherical coordinates, three-dimensional models of spheres and pictures of them are used. Especially tangent planes and intersections between planes and a sphere are considered.

Then they practice by considering exercises, including ones with real life applications.

Objectives: **What the students will know**:

- To identify corresponding intersecting planes or tangent planes to a given spherical triangle.
- To assign Euclidean triangles to spherical triangles in corresponding intersecting planes or tangent planes and identify the associated measures.
- To use Euclidean trigonometry to perform spherical calculations.

What the students will be able to do:

- To calculate the arc length between two points on a sphere using the cosine rule for sides.
- To solve real world problems.
- To state and prove the spherical cosine rule for sides.
- To develop skills for problem solving.
- To identify/ develop/ create applications of the related concepts and processes in the real world.

What attitudes the approach will foster:

- To develop critical thinking skills.
- To adopt various strategies for problem solving.
- To develop motives and positive affective tendencies for mathematics.

Materials/ Tools: Traditional board and geometrical equipment. Cardboard, paper, markers, scissors, glue, if possible transparent paper. Small balls and three elastic bands for every student.

Resources used by the teacher:

Spherical-geometry.pdf
[Spherical law of Cosine](https://www.youtube.com/watch?v=Y8VgvoEx7HY)
https://www.youtube.com/watch?v=Y8VgvoEx7HY

Resources for the student:

Spherical-geometry.pdf

Approaches/ Methodology: As a first step the students are asked to find and explore Euclidean triangles in intersecting planes through two edges of a spherical triangle and the centre of the sphere which correspond to the given spherical triangle. The students are then asked to specify measurements of angles and sides of the triangles found. The same is asked for Euclidean triangles in a plane tangent to the sphere through an edge of the spherical triangle.

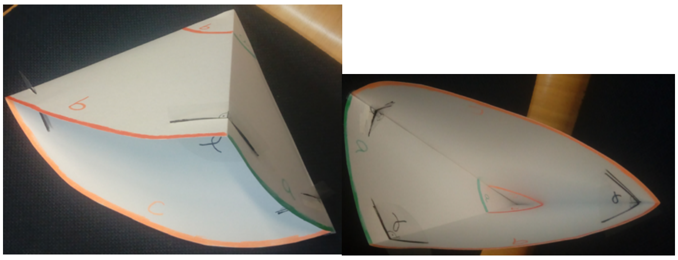
As a second step the students use this knowledge to calculate a missing side in a spherical triangle.

This procedure is generalised and provides the cosine rule for sides. Thus the students state the cosine rule for sides and prove it.

Next, students use their newly acquired knowledge to calculate distances between two arbitrary points of the sphere from given spherical coordinates.

Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
1 week earlier than the classroom consideration	STEP 1 **Preparatory work: Have a globe for demonstration in class. Make a large model of a spherical triangle including the centre of the sphere and the central angles from cardboard. Mark the centre angles there. Use transparent paper to mark the inner angles of the spherical triangle.**  In preparation, the students should make such a spherical triangle (smaller) out of paper, label it and bring it to the next lesson. An instruction for that is in the appendix.	

	Students recapitulate the contents of the spherical geometry learning plan from grades 7 to 9. On the basis of a problem the students learn that the existing knowledge is not sufficient to determine distances between any points on earth exactly (i.e. not experimentally).	

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
In the classroom on the planned day for the lesson	**Preparatory work: Have a globe for demonstration in class. Provide every student with a small ball and three elastic bands. Use the large model of a spherical triangle from above. Use transparent paper to demonstrate planes intersecting the sphere or tangent to the sphere.** Find and explore Euclidean triangles in intersecting planes through two edges of a spherical triangle and the centre of the sphere which correspond to the given spherical triangle. Use the paper models of spherical triangles prepared at home. Further use transparent paper to demonstrate planes. Make sketches of the found Euclidean triangles and determine the different measurements. Do the same for Euclidean triangles in a plane tangent to the sphere through an edge of the spherical triangle. Go back and forth between the sketches adding more and more measurements. Discuss the validity and the structure of their presentations Identify difficulties and weaknesses. Generalize by asking to specify : Derive a concrete formula. Specify which sizes are required. Ask them to state it and explain it between themselves. *Solution:* $\cos a = \cos b * \cos c + \sin b * \sin c * \cos \alpha$ *(For intermediate steps see spherical-geometry.pdf)*	Discussion How can the arc length of one side of a triangle be computed, if you know the arc lengths of the other sides and the enclosed angle?
	Generalise the results obtained and state a suitable theorem.	

	Solution: Cosine rule for sides *In a spherical triangle ΔABC with the measurement of the inner angles α, β, γ and the measurements of the centre angles a, b, c we have* *$\cos a = \cos b * \cos c + \sin b * \sin c * \cos \alpha$;* *$\cos b = \cos a * \cos c + \sin a * \sin c * \cos \beta$, and* *$\cos c = \cos a * \cos b + \sin a * \sin b * \cos \gamma$.* The proof found is then written down.	

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
In the classroom the next day	**Preparatory work: Have a globe for demonstration in class. Provide every student with a small ball and three elastic bands.** Ask the students to use their newly acquired knowledge to calculate distances between two arbitrary points of the sphere from given spherical coordinates. Here the students need to introduce several spherical triangles with one computed measurement leading to the next one.	Provide a globus so that the students can choose points on earth and their spherical coordinates.

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
	Proceed with the learning plan of the Galilean Moons	

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
	Provide material that will help in realizing the achievement of the objectives: multiple choice test covering both cognitive and affective domain issues Self-assessment	

Appendix

Instructions for making an open spherical triangle for students

Cut out a circle from paper, the center must be marked. Draw four radii that include different center angles (e.g. 50, 70, 100 and 140). Cut out one sector of the circle. Mark the arcs of the remaining circle sectors in different colors. Fold the paper along the radii and glue the two cut radii together. The colored circle arcs now form a spherical triangle. You can mark the inner angles of the spherical triangle by gluing strips of tape over two circle arcs right next to the corners of the triangle so that it does not deform. You can now write the name of the interior angle on the tape.