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"INNOMATH - Innovative enriching education processes for Mathematically Gifted Students in Europe" Project Number: 2019-1-DE03-KA201- 059604

Title of Content: Modul 3 – Discovery Based Learning

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Topic Derivation und modelling of constrained growth processes with the natural exponential function

Target group Students at grade 10/11





Content requirement

Basic understanding of simple exponential functions and the concept of derivatives (differential calculus)

Steps and contents

STEP o: Introductory STEP 1: Derivation of the (natural) exponential function STEP 2: Derivation rules for exponential functions STEP 3: Modelling constrained and logistic growth





STEP o: Introductory

How we can model this growths by functions?



CONSTRAINED GROWTH LOGISTIC GROWTH





STEP 1: Derivation of the (natural) exponential function

- Objective: Students discover the relationship between known exponential functions and their derivatives using Geogebra. They make assumptions about a special exponential function and finally derive Euler's number e using the differential quotient.
- Materials: worksheet page 1
 - geogebra file 1
 - youtube videos
 - Derivation and summary of Euler's number e:
 - https://www.youtube.com/watch?v=m2MIpDrF7Es
 - Excursus on further properties and connections of Euler's number e:
 - https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
 - **Processing time:** approx. 75 min (incl. videos)





STEP 2: Derivation rules for exponential functions

- **Objective:** Students repeat (known) simple derivation rules in the new context of exponential functions and learn the chain rule. They use these to form the derivative of functions of varying degrees of difficulty.
- Materials: worksheet page 2
 youtube video
 Proof of the chain rule

https://www.youtube.com/watch?v=moLZX19Dyyl

• Processing time: approx. 30 min (incl. video)





STEP 3: Modelling constrained and logistic growth

- **Objective:** Students analyse graphical representations and derive the equations for constrained and logistic growth functions. They work on several exercises (also in factual contexts) and give function equations, determine growth rates and learn about properties of the growth concepts.
- Materials: worksheet page 3
 - geogebra file 2
 - calculator
 - if available, but not necessary: computer algebra system (CAS)
- Processing time: approx. 75 min (incl. videos)





REFLECTION

Summary for modelling of constrained growth processes with exponential functions

TASK: Describe in your own words what you have learned from this learning module.

PADLET: https://kant-gymnasium.padlet.org/okaufmann/jmjwo7stw5mli2uy

CONTRIBUTION:

- written form as text
- auditorily as a voice message
- visually as a graphic
- audio-visually as a video





INNOMATH Consortium







