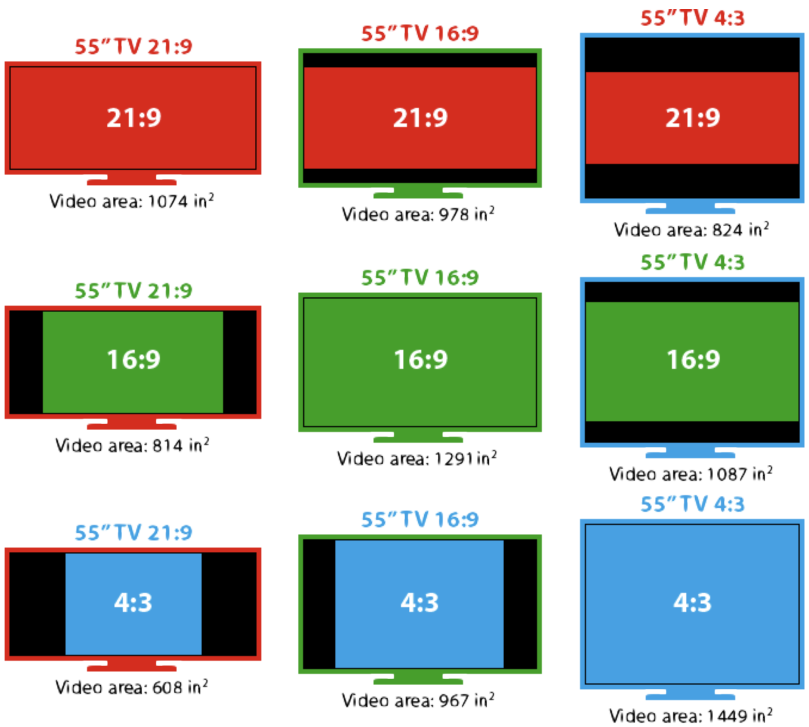
Ratio!

___**Question:** What is for you a ratio? What about 1:2?

Does a picture occur to you with the ratio 4:3 or 16:9 or 21:9=7:3?



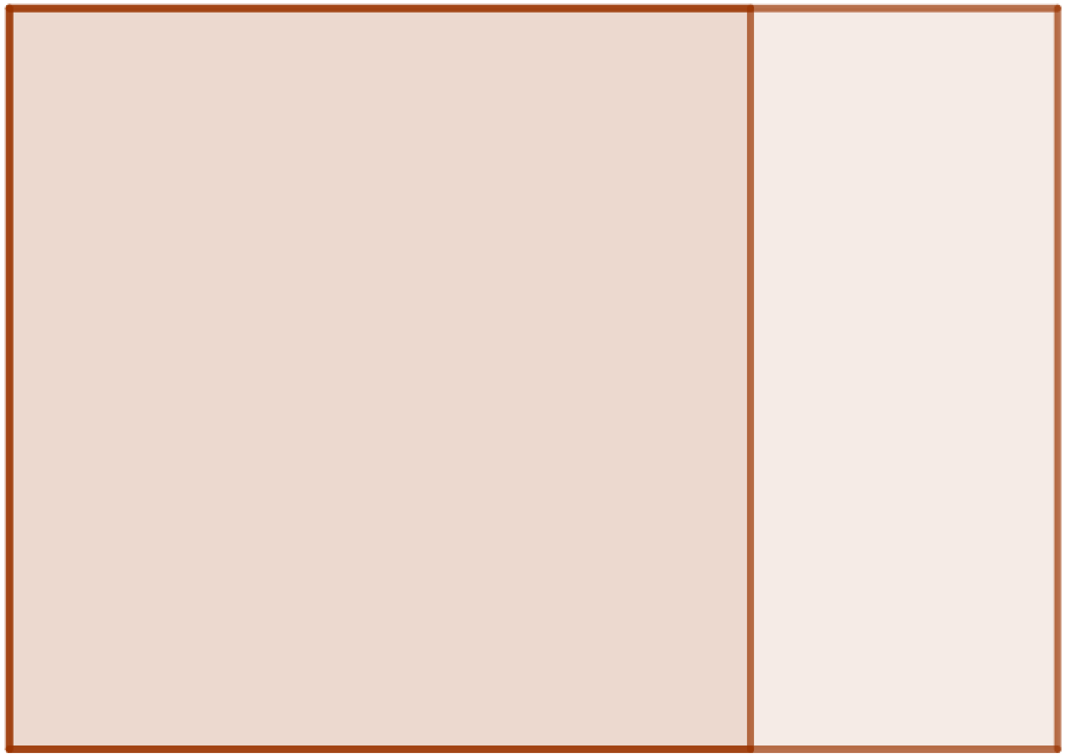
___**Conclusion:** A ratio is a number and it can be pictured as a rectangle.

___**Hands on:** Consider an A4 paper sheet. What do you know about it? What is its property?
___Do you know about A3 or A5 paper format? They actually have the same ratio!

We are going to investigate the ratio defining the (landscape) A4 paper as a number.

___**Subtract a square:**

Fold the left side of the sheet to the top in order to visualize a square. Remove it. What is the form of the remaining band of paper? What does it tell you regarding the ratio of the landscape A4 paper?



Conclusion:

______A ratio larger than one is associated with a wide rectangle, a ratio smaller than one is associated with a narrow rectangle.

______The ratio of the landscape A4 is greater than 1 but smaller than 2.

Now consider the remaining band of paper. It is associated with a number smaller than one. But rotating this narrow rectangle by a quarter of a turn, it becomes now a wide rectangle, associated with the *inverse number*, which is greater than one.

How many squares fit in there? What does it mean regarding the ratio of the landscape A4 rectangle?

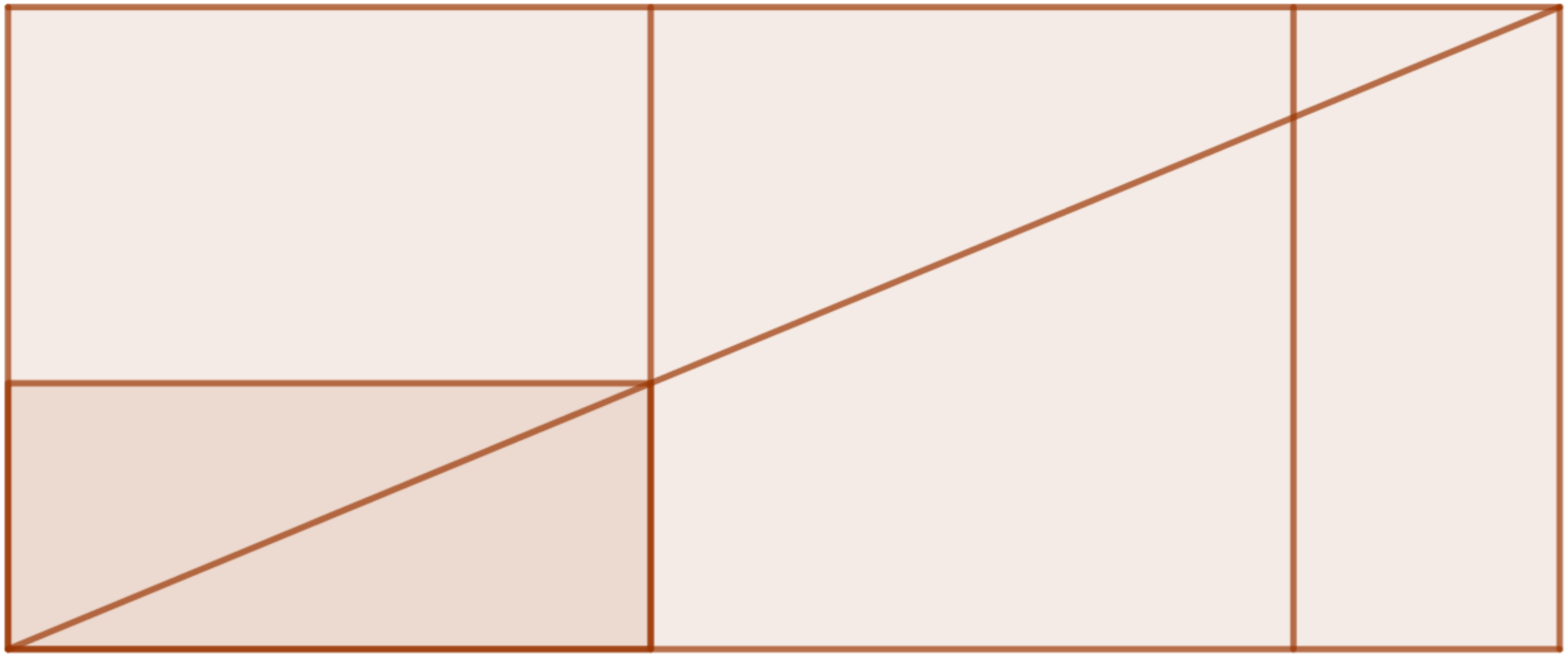
Let's call a the ratio of the landscape A4 rectangle. There are 2 squares in this long stripe and still a narrow stripe remains.

Conclusion: $1 + \frac{1}{3} < a < 1 + \frac{1}{2}$

Actually, this narrow stripe resembles the previous one, don't you think? Try to investigate.

______**Question:** How can you be sure, geometrically that two rectangles have the same ratio?

Conclusion: To check that two rectangles share the same ratio, pinch them at one corner and verify that their diagonals are the same.



It is the case for the A4 paper stripe. What does it tell you regarding the ratio a?

It tells that $1 + a = 2 + \frac{1}{1+a}$ that is to say $1 + a = 2 + \frac{1}{2+\frac{1}{2+\frac{1}{1+a}}} = 2 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\frac{1}{\dots}}}}$.

Question: can you try to figure out what is the value of $(1 + a)^2$ and a^2?

Another way to look at it through **enlargement**: The A4 paper has the same ratio has the A5 paper which is an A4 paper folded in two.

Let p and q be the length and width of the A4 paper.

Question: What are the dimensions of the A5 paper?

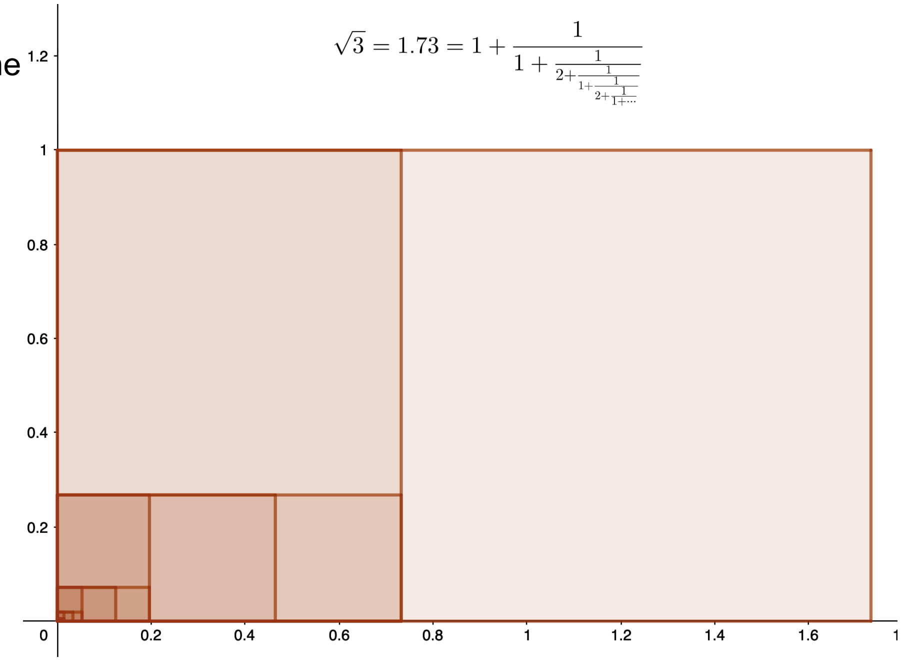
And what are their ratios?

The length and width of the A5 paper are then q and $\frac{p}{2}$, hence the ratio $a = p:q$. fulfils $a = p:q = q:\left(\frac{p}{2}\right) = \frac{2}{a}$. Therefore $a^2 = 2$.

Likewise, $\sqrt{3}$ is characterized as being the ratio of a rectangle similar to a third of it.

In fact the square root of an integer can be characterized this way, leading to the alternative: it is whether an integer or an irrational.

Link to the geogebra applet 1 and geogebra applet 2.



Origamis

Tetrahedra and octahedra.

Follow the instructions to build a paper of ration $\sqrt{3}:1$

Then use it to construct four tetrahedra and one octahedron.

Then use them to build a bigger tetrahedron.

What is the ratio of volume between the smaller and the larger tetrahedron?

What does it mean regarding the volume of the octahedron?

With your friends, unite to build a yet twice larger tetrahedron.