

A large geometric diagram consisting of two overlapping circles. The left circle is larger and has a horizontal dashed line passing through its center. The right circle is smaller and overlaps the right side of the larger circle. Several lines connect the centers of the circles and their intersection points. Labels 'B', 'X', 'C', 'Q', and 'D' are placed at various points: 'B' is near the top intersection, 'X' is at the intersection of the two circles, 'C' is on the right side of the smaller circle, 'Q' is at the bottom intersection, and 'D' is at the bottom of the larger circle.

**ELECTRONIC GUIDEBOOK
OF METHODS AND TOOLS
FOR TEACHER FACILITATORS**

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Electronic Guidebook of Methods and Tools for teacher facilitators

INNOMATH Guidebook Digital Portal (INNOMATH-GDP)

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INNOMATH O3: ELECTRONIC GUIDEBOOK OF METHODS AND TOOLS FOR TEACHER FACILITATORS

EXECUTIVE SUMMARY (EN)

The **Electronic Guidebook of Methods and Tools for teacher facilitators** is the Output 3 of the INNOMATH project. This Output contains the Output 1: Analysis Report on Good Practices and Methods used to support gifted/talented pupils in schools and the Output 2: Mathematics meets Industry in School – Knowledge to Innovation through Practice: Guidelines

This Guidebook provides an alternative method of teaching and learning mathematics, which is expected to contribute to the improvement of the understanding and appreciation to the applications and problem solving in industry through mathematics applications. Pupils and teachers will also develop transversal talent in communication skills for the teaching and learning of mathematics.

The part A of the Electronic Guidebook gathers good practices related to the education, development and stimulation of mathematically talented students and suggests a collection of definitions of a mathematically talented student.

The part B of the guidebook highlights good practices related to the education, development and stimulation of mathematically talented students by providing a collection of mathematics problems for supporting gifted/talented students separated into two age groups, under 14 and over 14 years old. The collection of good practices includes problems and applications, competitions and communication activities and related videos.

Through this Electronic Guidebook of Methods and Tools for teacher facilitators, industry will also be benefitted and be able to discover the creative thinking and potential of mathematically gifted pupils. A way to achieve this is through connecting the gifted students with the industry through a MID-Day (Mathematics in Industry Day). Therefore, the part C of the Guidebook provides the guidelines for planning and conducting a MID-day event.

INNOMATH O3: ELEKTRONISCHER LEITFADEN FÜR METHODEN UND WERKZEUGE FÜR LEHRERMODERATOREN

EXECUTIVE SUMMARY (DE)

Dieser elektronische Leitfaden mit Methoden und Werkzeugen für Lehrkräfte ist der dritte Output des INNOMATH-Projekts. Dieser Output enthält zum einen den Analysebericht über bewährte Praktiken und Methoden zur Förderung begabter SchülerInnen in Schulen und den Leitfaden *Mathematics meets Industry in School – Knowledge to Innovation through Practice: Guidelines* („Mathematik trifft auf Industrie – Wissen zu Innovation durch Praxis“). Er bietet alternative didaktische Methoden für das Lehren und Lernen im Fach Mathematik, die dazu beitragen sollen, das Verständnis und die Wertschätzung für die anwendungsorientierte und problemlösende Behandlung mathematischer Themen aus der Industrie zu fördern. Einen weiteren zentralen Aspekt stellt die Entwicklung transversaler Kommunikationsfähigkeiten der SchülerInnen und LehrerInnen dar.

Teil A des elektronischen Leitfadens umfasst bewährte Unterrichtspraktiken im Zusammenhang mit der Wissensvermittlung, Entwicklung und Förderung mathematisch begabter SchülerInnen und definiert den Begabungsbegriff.

Teil B des Leitfadens dient als Aufgabenpool, in dem eine Sammlung von Mathematikaufgaben für die Förderung von SchülerInnen mit mathematischer Hochbegabung bereitgestellt wird, welche sich an zwei Altersgruppen orientiert, nämlich Kinder unter 14 Jahren zum einen und über 14-Jährige zum anderen. Der Leitfaden umfasst Aufgaben und Umsetzungen, Anregungen für eine lernwirksame Gestaltung von Herausforderungen und Kommunikationsaktivitäten sowie entsprechende Videos.

Von diesem digitalen Leitfaden mit didaktischen Methoden und Werkzeugen für Lehrkräfte wird auch die Industrie profitieren und in der Lage sein, das kreative Denken und Potenzial mathematisch begabter SchülerInnen zu entdecken. Hierbei empfiehlt es sich zum Beispiel, diese im Rahmen eines MID-Day (Mathematics in Industry Day) mit der Industrie in Kontakt zu bringen und somit Theorie und Praxis zu verbinden. Teil C des Leitfadens enthält daher Leitlinien für die Planung und Durchführung eines solchen MID-Days.

INNOMATH 03: ΗΛΕΚΤΡΟΝΙΚΟΣ ΟΔΗΓΟΣ ΜΕΘΟΔΩΝ ΚΑΙ ΕΡΓΑΛΕΙΩΝ ΓΙΑ ΕΚΠΑΙΔΕΥΤΙΚΟΥΣ-ΔΗΜΙΟΥΡΓΟΙ ΓΝΩΣΗΣ

ΠΕΡΙΛΗΨΗ (GR)

Ο Ηλεκτρονικός Οδηγός Μεθόδων και Εργαλείων για εκπαιδευτικούς-δημιουργούς γνώσης (**Electronic Guidebook of Methods and Tools for teacher facilitators**) είναι το 3^ο πνευματικό παραδοτέο του ευρωπαϊκού έργου INNOMATH. Αυτό το παραδοτέο περιέχει το Πνευματικό παραδοτέο 1: Analysis Report on Good Practices and Methods (Εκθεση ανάλυσης καλών πρακτικών και μεθόδων) που χρησιμοποιούνται για την υποστήριξη χαρισματικών/ταλαντούχων μαθητών στα σχολεία και το Πνευματικό παραδοτέο 2: Mathematics meet Industry in School – Knowledge to Innovation through Practice: Guidelines (Τα μαθηματικά συναντούν τη βιομηχανία στο σχολείο – Γνώση στην καινοτομία μέσω της πρακτικής: Οδηγίες)

Αυτός ο οδηγός παρέχει μία εναλλακτική μέθοδο διδασκαλίας και εκμάθησης μαθηματικών, η οποία αναμένεται να συμβάλει στη βελτίωση της κατανόησης και της εκτίμησης των εφαρμογών και των προβλημάτων που επιλύονται στη βιομηχανία μέσω διάφορων εφαρμογών των μαθηματικών. Οι μαθητές και οι εκπαιδευτικοί-δημιουργοί γνώσης θα αναπτύξουν επίσης εγκάρσιο ταλέντο στις επικοινωνιακές δεξιότητες για τη διδασκαλία και τη μάθηση των μαθηματικών.

Το Μέρος Α του Ηλεκτρονικού Οδηγού συγκεντρώνει καλές πρακτικές που σχετίζονται με την εκπαίδευση, την ανάπτυξη και την διέγερση μαθηματικά ταλαντούχων μαθητών και προτείνει μια συλλογή ορισμών για ταλαντούχους μαθητές στα μαθηματικά.

Το μέρος Β του οδηγού υπογραμμίζει τις καλές πρακτικές που σχετίζονται με την εκπαίδευση, την ανάπτυξη και την διέγερση μαθηματικά ταλαντούχων μαθητών παρέχοντας μια συλλογή μαθηματικών προβλημάτων για την υποστήριξη χαρισματικών/ταλαντούχων μαθητών που χωρίζονται σε δύο ηλικιακές ομάδες, κάτω των 14 ετών και άνω των 14 ετών. Η συλλογή καλών πρακτικών περιλαμβάνει προβλήματα και εφαρμογές, διαγωνισμούς και δραστηριότητες επικοινωνίας καθώς επίσης και σχετικά βίντεο.

Μέσω αυτού του Ηλεκτρονικού Οδηγού Μεθόδων και Εργαλείων για εκπαιδευτικούς-δημιουργούς γνώσης, θα επωφεληθεί επίσης και η βιομηχανία καθώς θα μπορέσει να ανακαλύψει τη δημιουργική σκέψη και τις δυνατότητες των μαθηματικά ταλαντούχων μαθητών. Ένας τρόπος για να επιτευχθεί αυτό είναι η σύνδεση των χαρισματικών μαθητών με τον κλάδο μέσω μιας οργάνωσης MID-Day (Μαθηματικά στη Βιομηχανία μέρα). Ως εκ τούτου, το μέρος Γ του Οδηγού παρέχει τις κατευθυντήριες γραμμές για τον προγραμματισμό και τη διεξαγωγή μιας εκδήλωσης για τα Μαθηματικά στη Βιομηχανία Μέρα (MID-day).

INNOMATH O3: ELEKTRONICZNY PRZEWODNIK METOD I NARZĘDZI DLA WSPARCIA NAUCZYCIELI

PODSUMOWANIE WYKONAWCZE (PL)

Elektroniczny przewodnik po metodach i narzędziach dla wsparcia nauczycieli stanowi Rezultat 3 projektu INNOMATH. Ten Rezultat zawiera Rezultat 1: Raport z analizy dobrych praktyk i metod stosowanych w celu wspierania uzdolnionych/utalentowanych uczniów w szkołach oraz Rezultat 2: Matematyka spotyka przemysł w szkole – od wiedzy do innowacji poprzez praktykę: wytyczne.

Niniejszy Przewodnik przedstawia alternatywną metodę nauczania i uczenia się matematyki, która ma przyczynić się do poprawy zrozumienia i docenienia zastosowań i rozwiązywania problemów w przemyśle poprzez zastosowania matematyki. Uczniowie i nauczyciele rozwijają również przekrojowe umiejętności komunikacyjne w nauczaniu i uczeniu się matematyki.

Część A Elektronicznego Poradnika gromadzi dobre praktyki związane z kształceniem, rozwojem i aktywizacją uczniów uzdolnionych matematycznie oraz proponuje zbiór definicji ucznia uzdolnionego matematycznie.

W części B przewodnika podkreślono dobre praktyki związane z kształceniem, rozwojem i aktywizacją uczniów uzdolnionych matematycznie poprzez dostarczenie zbioru zadań matematycznych do wspierania uczniów uzdolnionych/uzdolnionych podzielonych na dwie grupy wiekowe, poniżej 14 i powyżej 14 roku życia. Zbiór dobrych praktyk obejmuje problemy i aplikacje, konkursy i działania komunikacyjne oraz powiązane filmy.

Dzięki temu Elektronicznemu Przewodnikowi Metod i Narzędzi dla wsparcia nauczycieli, przemysł również odniesie korzyści i będzie mógł odkryć kreatywne myślenie i potencjał uczniów uzdolnionych matematycznie. Sposobem na osiągnięcie tego jest połączenie utalentowanych studentów z branżą poprzez Dzień DMP (Dzień Matematyki w Przemysle). Dlatego w części C Przewodnika znajdują się wskazówki dotyczące planowania i prowadzenia imprezy w dniu DMP.

INNOMATH O3: Guide de méthodes et d'outils pour les enseignants facilitateurs

RESUME (FR)

Ce **Guide électronique de méthodes et d'outils pour les enseignants facilitateurs** est le résultat 3 du projet INNOMATH. Il se décompose en deux parties:

1 : Rapport d'analyse sur les bonnes pratiques et méthodes utilisées pour soutenir les élèves intellectuellement précoces dans les écoles et

2 : Les mathématiques rencontrent l'industrie à l'école – Des connaissances à l'innovation par la pratique : Lignes directrices

Ce guide propose une méthode alternative d'enseignement et d'apprentissage des mathématiques, qui devrait contribuer à l'amélioration de la compréhension et de l'appréciation des applications et de la résolution de problèmes dans l'industrie grâce aux applications mathématiques. Les élèves et les enseignants développeront également des talents transversaux dans les compétences de communication pour l'enseignement et l'apprentissage des mathématiques.

La partie A du Guide électronique rassemble les bonnes pratiques liées à l'éducation, au développement et à la stimulation des élèves doués en mathématiques et propose un ensemble de définitions d'un élève doué en mathématiques.

La partie B du guide met en évidence les bonnes pratiques liées à l'éducation, au développement et à la stimulation des élèves doués en mathématiques en fournissant une collection de problèmes mathématiques pour soutenir les élèves intellectuellement précoces séparés en deux groupes d'âge, moins de 14 ans et plus de 14 ans. La collection de bonnes pratiques comprend des problèmes et des applications, des concours et des activités de communication et des vidéos connexes.

Ce guide électronique de méthodes et d'outils pour les enseignants facilitateurs, peut également bénéficier au monde de l'industrie, mettant à profit la pensée créative et le potentiel des élèves doués en mathématiques. Un moyen d'y parvenir consiste à connecter les étudiants doués à l'industrie par le biais d'une journée Math-Industrie, un « MID-Day ». La partie C du Guide fournit les lignes directrices pour la planification et la conduite d'un tel événement.

INNOMATH O3: GHID ELECTRONIC DE METODE ȘI INSTRUMENTE PENTRU PROFESORI FACILITATORI

REZUMAT (RO)

Ghidul electronic de metode și instrumente pentru profesori facilitatori este Produsul Intelectual 3 al proiectului INNOMATH. Acest rezultat conține Produsul Intelectual 3: Raport de analiză a bunelor practici și metode utilizate pentru a sprijini elevii supradotați/talentați în școli și Produsul Intelectual 2: Matematica întâlnește industria în școală – De la cunoaștere la inovare prin practică: Îndrumări

Acest Ghid oferă o metodă alternativă de predare și învățare a matematicii, metodă de la care se așteaptă să contribuie la îmbunătățirea înțelegerii și aprecierii aplicațiilor și soluționării problemelor din industrie prin aplicații de matematică. Elevii și profesorii vor dezvolta, de asemenea, un talent transversal în abilitățile de comunicare pentru predarea și învățarea matematicii.

Partea A a Ghidului electronic colectează bune practici legate de educația, dezvoltarea și stimularea elevilor talentați la matematică și sugerează o colecție de definiții ale unui elev talentat la matematică.

Partea B a ghidului evidențiază bunele practici legate de educația, dezvoltarea și stimularea elevilor talentați la matematică, oferind o colecție de probleme de matematică pentru sprijinirea elevilor supradotați/talentați, împărțiți în două grupe de vârstă, sub 14 ani și peste 14 ani. Colecția de bune practici include probleme și aplicații, competiții și activități de comunicare și videoclipuri aferente.

Prin acest Ghid electronic de metode și instrumente pentru profesorii facilitatori, industria va avea de asemenea beneficii și va putea descoperi gândirea creativă și potențialul elevilor dotați din punct de vedere matematic. O modalitate de a realiza acest lucru este conectarea studenților/elevilor dotați cu industria prin Evenimentul Matematica întâlnește Industria (MID-Day). Prin urmare, partea C a Ghidului oferă îndrumări pentru planificarea și desfășurarea unui eveniment MID-Day.

Introduction

This **Electronic Guidebook of Methods and Tools for teacher facilitators** is the Output 3 of the INNOMATH project. This Output contains the Output 1: Analysis Report on Good Practices and Methods used to support gifted/talented pupils in schools and the Output 2: Mathematics meets Industry in School – Knowledge to Innovation through Practice: Guidelines

This Guidebook will provide an alternative method of teaching and learning mathematics, which is expected to contribute to the improvement of the understanding and appreciation to the applications and problems solving in industry through mathematics applications. Industry will also discover the creative thinking and potential of mathematically gifted pupils. Pupils and teachers will also develop transversal talent in communication skills for the teaching and learning of mathematics.

Part A: Definitions/characteristics of giftedness:

Foreword

The main task of this chapter is to gather good practices related to the education, development and stimulation of mathematically talented students, as well as to collect the definition of a mathematically talented student. To this end, a structure was designed to facilitate the presentation of the above practices and definitions. In this chapter, we've gathered definitions describing talented/above average mathematically endowed students. They are to help mathematics teachers working with students aged 10-18 to recognize students who are particularly talented in mathematics among their students. The chapter is divided into a theoretical part in which we collect the definition and two parts in which we approach the matter in an operational manner. We collect a set of traits and behaviors that a student should be characterized so that we can qualify him as a talented student. In the last part, we present a few methods that a teacher can use in his work to assess which of his/her students manifests mentioned traits and behaviors.

Set of definitions for gifted/talented school students

- **Gifted / talented student** - the student with an above average intellectual capacity (at the psychometric level, over 130), observing cognitive differences both at the quantitative and qualitative level, a higher maturity (perception and visual memory), capacity development metacognitive at an early age (about 6 years) and intuition in solving problems
- **Gifted / talented student** - the student with an intellectual curiosity, an extraordinary capacity for observation
- **Gifted / talented student** - the student who has an exceptional spontaneity in the school and demonstrates a high ability in academic work
- **Gifted / talented student** -students with creative thinking skills, students with original and divergent ideas, who demonstrate their skills in developing original ideas, being able to distinguish a given situation from several different forms
- **Gifted / talented student** - the student with visual and performing arts skills: this category includes those who have superior skills for painting, sculpture, drawing, dance, vocal and instrumental music and theatre
- **Gifted / talented students** - An intellectual, artistic, or leadership student who can achieve in academic fields and who needs differentiated and challenging activities in order to accomplish and reach his full potential both academically and personally.
- **Gifted / talented students** - An individual who demonstrates outstanding levels of aptitude (defined as an exceptionality to reason and learn) or competence (documented performance or achievement and in the top 10%) in one or more subjects
- **Gifted / talented students** - One-dimensional definitions of giftedness are not helpful. Children are neither gifted by external influences nor are they gifted per se. From a dialectical perspective development is always both an actual component and an objective one. In other words: one is already mature as a toddler, but at the same time the child always needs the “friendly guidance of adults”, in order to achieve that critical-constructive socialization which allows people to become responsible citizens. Therefore, we do not see talent as a manifestation of performance per se, but as a disposition for high and excellent performance. Kindergarten, school, and the leisure sector provide many opportunities to take up these individual dispositions and, in the best case, transform them into performance. Gifts are specific. The construct of multiple intelligences represents one of several modern models of

giftedness and should form the basis of our support. The approach is based on the assumption that intelligence is composed by many individual abilities, which in course of a development and learning process are combined to form higher-level abilities. This means that adolescents are not equally capable of high performance in all areas of intelligence, their talents can become manifest in a domain-specific way. The support of giftedness must therefore be multi-perspective and take place in different spheres of life.

- **Gifted / talented students** are children who are said to be thanks outstanding abilities are able to demonstrate advanced achievements. These are children with achievements and / or potential abilities in one or more areas, such as: general mental abilities, specific skills in the main school subjects, leadership skills, fine arts and performance, psychomotor skills.
- **Gifted / talented students** are children who demonstrate opportunities for advanced achievement in the field of mental, creative, artistic, leadership skills or in specific subjects and who, in order to fully develop these opportunities, require services or activities not provided by a standard school
- **Gifted / talented students** are characterized by accelerated psychomotor development, cognitive hunger manifested in the form of versatile interests, learning about new phenomena and situations, finding new tasks, as well as frequently asking himself various philosophical and existential questions, which are often problematic for parents and teachers. Large possibilities in the field of memorizing, storing and reproducing information allow a student capable of acquiring and accumulating more and more knowledge about the surrounding world, which pushes the able child towards increased intellectual activity in the form of the ability to conceptualize, abstract and synthesize and perceive the causal relationship. -effective.
- **Gifted / talented students** are one who have the ability to achieve outstanding achievements in a given field, who displays a high level of general abilities (intelligence) or have some special ability in the sphere of mental activity, for example in mathematics. A gifted student is also a student with a high or outstanding IQ.
- **A gifted / talented student**, in socially understood, it is usually a student who has exemplary behavior and the highest grades in school subjects. In the psychological sense, a gifted student is one who has a high IQ, high achievements, feels cognitive anxiety, is able to break away from the usual patterns, is able to find himself in a new situation, has ideas for new solutions to old problems, is not afraid of new things.

Characteristics-Executive Functions of gifted students

Some features that can give important suggestions for discovering talented individuals in mathematics are the following:

1. An unusually vivid attention and intense curiosity about mathematics.
2. An unusual speed in learning, understanding and applying mathematical ideas.
3. A great ability to think and work abstractly and the ability to see patterns and relationships in a mathematical way.
4. An unusual ability to think and work with mathematical problems in more ways soon flexible, creative, rather than stereotypical.
5. An unusual ability to move from learning to new, unpredicted situations.

They differ from the general group of students and by the following abilities:

- Spontaneous configuration of problems;
- Flexibility in handling information; mental speed of fluency of ideas;
- Ability to organize information; originality of interpretation; the ability to transfer ideas;
- The ability to generalize;

Talented students in mathematics can have high levels of abstraction and have strong skills critical thinking. They are likely to quickly understand mathematical ideas and choose a creative approach to solve problems.

A gifted student might be one who:

- Ignores schoolwork but spends huge amounts of time pursuing his own topics of;
- Interest to a very advanced level;
- Learns new ideas or skills more rapidly than others;
- Can control their attention better than others;
- Commits entirely to a task that captures their imagination;
- Actively wants to develop that area in which they are gifted;
- Can immediately see multiple angles of perception of the same topic, object or idea;

Renzulli ¹defined gifted behaviors ²as those which result from the interaction between above average ability, task commitment, and creativity individuals who can manifest and apply this complex of traits to any potentially valuable area of human performance would be categorized as gifted.

Frasier and Passow's ³common ⁴traits of giftedness (1994⁵)

- Motivation;
- Communication skills;
- Well-developed memory;
- Insight;
- Imagination/creativity;
- Advanced ability to deal with symbol systems;
- Advanced interests;
- Problem-solving ability;
- Inquiry;
- Reasoning;
- Sense of humor.

¹ Joseph S. Renzuli – Director The National Research Center on the Gifted and Talented, University of Connecticut Storrs, Connecticut (USA).

² Renzulli, J. S., *What is this thing called giftedness, and how do we develop it? A twenty-five year perspective*, *Journal for the Education of the Gifted*. 23(1), 1999, 3-54.

³ *Mary M. Frasier (1938–2005) - was a famous African American educator who specialized in the area of gifted education at the University of Georgia.*

⁴ *A. Harry Passow (1920-1996) – Jacob H. Schiff Professor of Education, he was a one of the world's leading experts on both urban education and gifted education (Teachers College, Columbia University).*

⁵ Frasier M. M., Passow A. H., *Toward a new paradigm for identifying talent potential*, Research Monograph 94112, Storrs, CT: The National Research Center on the Gifted and Talented, University of Connecticut.

US Department of Education stresses:

- Performing or showing the potential to perform at a remarkably high level of accomplishment compared with others of their age, experience or background;
- High performance capability in intellectual/creative/artistic areas;
- Exceptional leadership abilities;
- Excellence in particular fields of interest.

Children capable of high performance include those with demonstrated achievement and/or potential ability in any of the following areas:

- General intellectual ability;
- Specific academic ability;
- Creative or productive thinking;
- Leadership ability;
- Visual and performing arts;
- Psychomotoric ability.

Five categories of definitions of gifts and talents are outlined:

- After-the-fact definitions emphasise prominence and outstanding achievement;
- IQ definitions set a point on the IQ scale and persons scoring above that point are classified as gifted (usually two standard deviations above the mean is the cutting point);
- Percentage definitions set a fixed proportion of the school (or district) as „gifted“, based on ability scores or grades;
- Talent definitions focus on students that are outstanding in art, music, sciences, or other specific aesthetic or academic areas;
- Creative definitions stress the significance of superior creative abilities.

Sternberg ⁶describes ⁷an implicit theory that summarizes “what we mean by giftedness people’s conception of giftedness”. Five necessary and sufficient conditions that gifted persons have in common:

- Excellence – a gifted person should be extremely good in something;
- Rarity – he/she must possess a high level of an attribute that is uncommon relative to peers;

⁶ Robert J. Sternberg - an American [psychologist](#) and [psychometrician](#), Professor of Human Development at [Cornell University](#).

⁷ Sternberg R. J., *The Concept of ‘Giftedness’: A Pentagonal Implicit Theory. The Origins and Development of High Ability*, CIBA Foundation Symposium 178, John Wiley & Sons, 1993, 5-16.

- Productivity – the superior trait must (potentially) lead to productivity;
- Demonstrability – the trait must be demonstrable;
- Value – the superior performance must be in an area that is valued by society.

Tools for Identification of gifted/talented school students

- A. Teachers and parents are the best people to identify gifted and talented students. Cognitive and affective characteristics of gifted and talented students which may differ from their classmates. Just as it is important to identify students with learning disabilities and assess their particular learning needs on the learning continuum, it is also necessary to identify each gifted child's specific learning needs and current level of achievement.
- B. There are **Academic Potential Tests like the Raven's Progressive Matrices assessment** which does not need professional administration and which is said to be able to identify underachievers with high potential. A Raven's Progressive Matrices Test is a test designed to measure your non-verbal, abstract and cognitive functioning. In the test, a candidate is presented with a matrix of 3x3 geometric designs, with one piece missing. The candidate's job is to choose the right diagram, from a set of eight answers that completes a pattern in the matrix that you have to figure out. The questions and answers are all completely non-verbal.
- C. **The Raven's Progressive Matrices Test** was developed in the 1930's by J.C. Raven to research how genetic aspects and environmental aspects influence intelligence. The Raven's Progressive Matrices Test is designed to have no cultural or ethnic bias, so it should measure only the genetic component of intelligence without the influence of environment. The relevant pdf file with more details is "Raven Progressive Matrices Test".
- D. There are also internal school tests, which are usually used as a kind of first step to identify those students that are quite easily identifiable. In practice, these tools work best when part of an identification matrix that takes into account a range of informal identification tools. For an idea of what this matrix might look like, here is the one test advised by Joseph Renzulli, one of the leading names in Gifted education in the US. Renzulli's method, presents the three-ring conception of giftedness. A detailed process is presented illustrating how students can be effectively screened and identified as gifted and talented through the three-ring conception approach. The relevant pdf file with more details is "Renzulli Test (Gifted and Talented)".
- E. There are two formulated tests, run by Johns Hopkins called the School and College Ability Test (SCAT) and the Spatial Test Battery (STB). These tests measure verbal, quantitative, and spatial reasoning ability. These are what are called Out of Level tests which have no ceiling

effect as they expose students to questions far beyond their level so they eventually hit a level they can't fulfil or exceed. As far as SCAT is concerned, the Mathematics section measures a student's understanding of fundamental number operations. The STB consists of four sections: Visual Memory Learning, Surface Development, Block Rotations and Visual Memory Recall. The relevant pdf file with more details is "SCAT & STB".

- F. Wechsler Intelligence Scale for Children (WISC), developed by David Wechsler, is an individually administered intelligence test for children between the ages of 6 and 16. It can be completed without reading or writing. The WISC takes 65–80 minutes to administer and generates an IQ score, which represents a child's general cognitive ability. This test assists teachers or specialists to discover learning strengths and vulnerabilities in each student; therefore, it allows them to offer targeted, individual intervention that is best suited for each student.

- G. The Intelligence Quotient (IQ) is a parameter for assessing general intelligence or evaluating specific factors of intelligence. This is done by a test. So, the IQ can only refer to it, because there is no scientifically recognized and unambiguous definition of intelligence. The IQ determined by the test is compared to a reference group. Such a group can be age-specific (e.g. children and adolescents) or specific to educational levels (e.g. high school). The IQ scale is based on the assumption of a normal distribution with a mean of 100 and a standard deviation of 15. Accordingly, about 68 % of the people in this reference group have an IQ in the middle range between 85 and 115. People with an intelligence quotient of at least 130 are described as gifted, because according to the normal distribution they comprise only about 2.3 % of the reference group. You can find further information and some IQ tests in the related pdf with the name "Intelligence Quotient".

- H. The tool for identification of gifted/talented school students developed by Humboldt University in cooperation with the "Berliner Netzwerk mathematisch-naturwissenschaftlich profilierter Schulen" is a method method for identification of giftedness comprising three parts. The points accumulated in the three parts are added and the school students performing best are accepted into the "Network".

1. Mathematical test (provides 50% of the whole identification test): School students take a mathematical test (75 minutes) with problems which are similar to the ones provided in the Kangourou Competition. For this test a school student can achieve at most 10 points.
 2. Grades from school (provide 25% of the whole identification test): The grades (1 very good, 2 good, 3 satisfactory, 4 sufficient, 5 unsatisfactory, 6 not sufficient) from the last report of the school are added and translated into points in the following way: Mathematics grade multiplied by 3 + German language grade multiplied by 2 + foreign language grade + natural sciences grade (this sum has to be at most 15).
 3. Special skills (provide 25% of the whole identification test): Each of the skills below, which is rated as particularly strong, yields a point. When all of the skills below are rated higher than average, a further point is awarded.
 - a. Recognizes basic principles or rules and applies them appropriately
 - b. Works in a structured, independent manner and links knowledge elements
 - c. Plans and organizes work steps purposefully and quickly is full of ideas, open to new things and interested in many different areas
- I. The tool for identifying gifted students is the questionnaire, the basic tool used to collect data. Its elaboration has a multidisciplinary character, in the sense that it requires knowledge from several scientific disciplines: economics, statistics, mathematics, sociology, psychology, etc.

The questionnaire must be designed so that the reading of the questions, the observance of the instructions and the recording of the answers are as simple as possible, both for the respondent and for the teacher. After selecting the questions, the format of the questionnaire will be established: the appearance, the length and the way the questions will flow.

To identify talented students in mathematics, teachers use three types of questionnaires.

1. The first **questionnaire** is about **the lifestyle of teenagers**. This will allow teachers to learn more about how their students live, feel and think. The share of this questionnaire in shaping the profile of the talented student is **30%**.

2. The **second questionnaire** focuses on **the attitude of students towards Mathematics**, because we aim to identify students who want to perform in the subject of mathematics. The share of this questionnaire in outlining the profile of the talented student is **30%**.

3. Finally, the third and most interesting questionnaire aims to allow students (and teachers) to discover **the dominant types of intelligence**, something that will allow us all to improve the teaching and learning of mathematics for talented students.

The third type of questionnaire is developed based on Howard Gardner's Theory of Multiple Intelligences, which says that the individual is a unique amalgam of skills and talents, and memory or logic are not the only elements that define us. According to Gardner's theory, there are 8 types of intelligences: Visual-spatial intelligence, Linguistic intelligence, Logical-mathematical intelligence, Body-kinesthetic intelligence, Musical intelligence, Interpersonal intelligence, Intrapersonal intelligence, Naturalistic intelligence.

By applying the third questionnaire we aim to identify students who have a superior logical-mathematical intelligence, having pattern recognition skills, deduction and logical analysis of problems. For them, numbers, relationships and patterns turn into concepts.

Characteristics:

- **Increased problem-solving ability**
- **Inclination to abstract thinking**
- **Pleasure for scientific experiments**
- **Talent in solving complex problems**

The share of this questionnaire in outlining the profile of the talented student is 40%.

At the school level, **the Register of talented and gifted students in mathematics** is established.

Find below more information about identifying talented students in math by using the three types of questionnaires mentioned in the summary.

- J. **The DISCOVER Projects** (Discovering Intellectual Strengths and Capabilities while Observing Varied Ethnic Responses) began in 1987 under the direction of Dr. C. June Maker at the University of Arizona. At the time, Dr. Maker had been analyzing various new theories of intelligence, the most notable of which was Howard Gardner's Theory of Multiple

Intelligences. She also had been studying groups of gifted children, as well as successful scientists who had overcome disabilities, to isolate factors contributing to exceptional success. She eventually determined that the most important component of exceptional success was the superior ability to solve complex problems. The DISCOVER Projects were created to study, categorize, and measure a broad spectrum of “problem solving strategies” used by various age groups of differing ethnic, economic, and cultural backgrounds.

- K. **Eby Primary School Recognition Tool. Teacher's Recommendation Form J. W. Eby** - The construction of the sheet is based on the definition of Renzelli's ability. It consists of three rooms: Abilities, Creativity, and Task Engagement. It consists of 15 items, 5 items for each of the scales. The teacher's task is to assess the test items according to the student's behaviour, which he makes by marking the appropriate item on a 4-point scale, on its basis he calculates the total score.

- L. **Eby index of behaviour indicative of giftedness. General list of features of J. W. Eby** - The worksheet consists of a list of student behavior that the teacher may observe during various school situations. The theoretical basis of the tool is Gardner's concept of Multiple Intelligence. A separate form has been constructed for each of the six types of giftedness: linguistic, mathematical, natural and problem-solving, social and leadership, visual-spatial, musical and technical orientation. Within each of them, the presence of ten behaviors can be verified: perceptiveness, active interactions with the environment, reflectivity, persistence, independence, goal-orientation, originality, efficiency, self-esteem and effective communicating ideas. Each of them has two positions in the sheet. The teacher assesses on a 5-point scale.

- M. **David Lewis Parenting Test** - The sheet consists of 35 items describing the characteristics of a gifted child. The parent marks the behaviors and features that he recognizes in his child: a gifted child, Intellectual Skills, Leadership Abilities, Artistic Skills, Musical Abilities.

- N. **Teacher's questionnaire for observation of student Layock's skills** - The tool is used to diagnose the level of a student's skills in ten aspects: language skills, drawing conclusions, asking questions, originality of ideas, problem solving, pace of thinking, imagination, memory, observation, concentration of attention. They are assessed using a five-point scale: "poor" - "exceptionally good".

- O. **Recognition check kits scientific, artistic, creative talent, linguistic, mathematical, sports, leadership and organizational talent, G. Lewis** - The sets include 12-28 statements presenting the child's behaviour. Person the diagnosing person marks the frequency of the behavior (1-never, 5-always). The evaluation is constructed on the basis of the number of points scored, the higher the number, the greater the likelihood of a student having a talent.
- P. **Scale of Creative and Imitative Attitudes (SPTO) R. M. Sigva** - The worksheet is used to diagnose the level of creative attitudes and identify motivation features. The tool consists of two modules: identification and self- description. The first one includes 4 elements of creativity: divergent thinking, motivation, elaboration skills and attitude to failure in creative activity. The second of these include strategies of students' coping with difficult situations, both in the Inter- and intrapersonal dimension.
- Q. **H. Gardner's Multiple Integration Questionnaire** - The questionnaire consists of 28 statements covering the following areas: Linguistic, Mathematical-logical, Visual-spatial, Music, Interpersonal, Intrapersonal, Kinesthetic. The respondent performs self-assessment by marking a grade from 0 to 5 with each statement. The sum of points and marking it on the diagram determines the profile of the dominant type / types of intelligence.

Part B: A collection of Good Practices and Methods used to support gifted/talented pupils in schools

Foreword:

This part of the Guidelines gathers good practices related to the education, development and stimulation of mathematically talented students by providing a collection of mathematics problem for supporting gifted/talented students separated into two age groups, under 14 and over 14 years old. The collection includes problems and applications, competitions and communication activities and related videos. To this end, a structure was designed to facilitate the presentation of the above material by providing a short abstract description following by a file reference that can be found with the same name in the link google drive file given in the link here below. For better access the reader is invited to copy the link and paste it into the URL to open the folder.

<https://drive.google.com/drive/folders/1LQAkjsh9F-n9iVa7V7FggIHOFDM3C7rh>

Chapter 1: A collection of Mathematics Problems for gifted/talented school students (age under 14)

In this chapter we've gathered examples of problems dedicated to talented students who are under 14. Among them you can find issues from many fields of mathematics, often related to other disciplines or topics encountered in everyday life. We wanted to show problems dedicated to students with special mathematical talent in a wide range of applications. All materials contained in this chapter have been collected and sent by Partners. They were selected on the basis of criteria established and approved during MEETING 1. This collection will select innovative practices and models that the user can decide what best fit its environment and culture and decide what can work and what cannot. This approach is mainly the innovative character that the project. The main reason for choosing that practice was: to encourage to think, to control the level of comprehension, knowledge, abilities, to focus on the task, to remind and repeat, to enable students to express their feelings, involvement, opinions, to provide an opportunity to answer questions, to encourage shy students to take part in the discussion, to teach by using students' answers, to ask gifted students as the way to stimulate others. We tried to make the found materials fit into previously prepared categories: concern one particular field, concern more than one field, concern one particular ability, concern more than one ability, serve as a revision, serve as an ability practice, bring new knowledge for the student, develop new abilities, building self-confidence. Topics of these problems include:

B1.1 The material concerns work in the field of geometry. Students have to use mainly the Pythagorean Theorem. There are several approaches possible. Students can start working directly at the problem and find solutions by themselves. Or they can use the given answers, of which exactly one is not correct, and deduce which is the wrong one. So it provides an excellent context for reflective thinking and deductive reasoning abilities. The complete solution is given.

B1.1.1 Geometry-Snowman.pdf

B1.2 The material concerns work in the field of stochastics and testing. Students might to use binomial distribution. However, they might as well use intuitive approaches. There are several approaches possible. Students can start working directly at the problem and find solutions by themselves. Or they can use the given answers, of which exactly one is not correct, and deduce which is the wrong one. So, it provides an excellent context for reflective thinking and deductive reasoning abilities. The complete solution is given.

B1.2.1 Stochastics-Fairy-light.pdf

B1.3 The material concerns work in the field of geometry and packing problems. Students might use intuitive sketches to get first ideas and confirm the ideas by the Pythagorean theorem. So, it provides an excellent context for straightedge and compass constructions joined by deductive reasoning abilities. The complete solution is given.

B1.3.1 Packing-problem.pdf

B1.4 The problem concerns the work in the field of geometry. A problem-centred approach was used to identify data and mathematical relationships. Completing the learning unit "Triangle Similarity" aims to provide the student with skills that allow him to: a) express ideas about similarity in the vocabulary of proportions b) application of results-fundamental theorem of similarity, similarity criteria, bisector theorem-in solving of practical problems. Students will acquire the skills necessary to identify pairs of similar triangles in given geometric configurations and will establish similarity relations between two triangles by different methods. The teacher will interpret the similarity of triangles in correlation with qualitative and / or metric properties and will apply the similarity of triangles and the bisector theorem in solving mathematical or practical problems.

B1.5 The problem is in the field of geometry. A problem-centered approach was used to know and understand the concepts, terminology and calculation procedures specific to the centre of gravity of the triangle and the medians. The purpose of completing the learning unit is to develop the students' ability to exploit / investigate the properties of the medians and the center of gravity in a triangle. The teacher will facilitate the development of the students' work capacity, using the mathematical language specific to the problems in which the cases of congruence of triangles are used. It is essential to develop interest and motivation for the study and application of geometry in various contexts, especially for talented students and especially interested in geometry. Students can pose challenging problems and work independently or in groups.

B1.5.1 The center of gravity of a triangle.pdf

B1.6 The problem is algebra, the divisibility chapter. In solving this problem, students are instructed to use the knowledge gained in the study of calculus with powers of natural numbers and the use of the last digit of the power of a natural number. The teacher points out that the last digit of the powers of 2 and 3 is repeated 4 by 4 and therefore the natural numbers m and n are considered in a certain form. By solving problems of this type, students will acquire the skills necessary to identify the criteria of divisibility in the multitude of natural numbers. It is essential to be aware at the level of students talented in mathematics, the fact that understanding the concepts, terminology and calculation procedures specific to the

criteria of divisibility is important in solving such problems. Students can ask challenging questions for further study and can work in groups that include more students interested in the arithmetic of natural numbers.

B1.6.1 Divisibility.pdf

B1.7 The MATHEU Volume 2 is a set of mathematics problems for gifted students under the age of 14 and they are designed in a latter form of difficulty. The problems are the outputs of the EU funded project MATHEU. The design is made so the higher the level reached the more talented the student is. At the same time the climbing of the latter serves as a motivation to students to reach higher level. In this manual the reader can find problem covering the following Mathematics topics: Dirichlet Principle, Mathematical Games, Geometry, Inequalities, Invariants, Number Theory, Polynomials and Equations, Arithmetic, Geometric and Harmonic Sequences.

B1.7.1 MATHEU-Volume2.pdf

B1.8 Problem 1 deals with the notion of symmetry, the properties of straight lines and menstruation in polygons. Working through this problem students, have to use their quantitative reasoning and apply their understanding of the notions of space, the connections between them and use geometric properties in real world objects.

B1.8.1 Problem 1 with solution.pdf.

B1.9 Problem 2 deals with the fields of geometry and trigonometry. This problem is a good example to be used for students to develop heuristic strategies in approaching the solution of a problem as there are many methods to work through the solution. Students will need to be able to extract important information from the problem, apply their mathematical reasoning and develop conceptual understanding, recalling and applying geometrical knowledge accurately to the properties of shapes. By being exposed to such problems, students develop their confidence, persistence and organization in problem solving.

B1.9.1 Problem 2 with solution.pdf

B1.10 Problem 3 deals with the properties of numbers and in particular prime numbers. This is an excellent example to train students to be able to formulate their mathematical thinking in mathematical sentences. Students will need to use the properties of operations to express numbers in different forms to assist in solving a problem. Students should be able to reflect on their solution and use their critical thinking to evaluate how rational their answer is based on the given facts.

B1.10.1 Problem 3 with solution.pdf.

B1.11 The material contains a problem of logical conclusions. Problem-solving-abilities are developed by processing given statements, so the students have to find suitable approaches (e.g., tables, hypotheses ...). It provides an excellent context for developing intuition, logical thinking and deductive reasoning abilities.

B1.11.1 Einstein's Riddle.pdf

B1.12 The material concerns problems in the field of geometry and functional relation. A problem-centered approach will help to develop problem-solving abilities, structured thinking, visual thinking and so on. Students can find challenging problems and work independently. The tasks provide insights into real problems that can be solved with mathematical tools.

B1.12.1 Challenging Problems.pdf

B1.13 The materials are a complete proposal for teaching mathematics with a graphic calculator and computer for students aged 12-14. It takes into account the learning objectives, teacher and school tasks and the content of mathematics teaching. Presented Mathematics, Calculators and Computer program, whose general assumption is that during the process of teaching mathematics, i.e., in the process of shaping mathematical concepts, conducting mathematical reasoning, solving tasks and problems and teaching mathematical language, we use computer and calculators at these moments, in which the teacher encounters a variety of problems that are difficult to overcome using traditional teaching methods and teaching aids used so far. In this concept, the computer and calculators are treated as an important didactic means, as an important help in achieving some of the intended goals.

B1.13.1 Matematyka Kalkulatory i Komputery.pdf

B1.14 Materials focus on teaching mathematics to students aged 6-10. This is a complete guide, which shows what are the characteristics of children at a younger school age and how to plan your work, taking into account the development opportunities of children, to comprehensively familiarize them with the world of mathematics. Many of the proposals presented create situations in which students that are interested and mathematically gifted try to find other non-standard solutions to problems, proving that they follow their own reasoning. Mathematical exercises and tasks collected do not inhibit the development of a student with extraordinary abilities but allow him to think independently and custom action. They encourage active participation in classes and creative approach to solved tasks.

B1.14.1 Rozwijanie_zainteresowan_i_zdolnosci_matematycznych-6-10.pdf

Chapter 2: A collection of Mathematics Problems for gifted/talented school students (age over 14)

In this chapter we've gathered examples of problems dedicated to talented students who are over 14. Among them you can find issues from many fields of mathematics, often related to other disciplines or topics encountered in everyday life. We wanted to show problems dedicated to students with special mathematical talent in a wide range of applications. As in the previous chapter, the criteria and rules for searching materials were established during Meeting 1. The questions that drove the search concerned: What kind of practices (methods or materials) is it? Why can teachers and students be interested in these materials? What are the most important reasons for choosing this material? What problems can I solve with these materials? Answers to what questions will be found? What is the main idea in the proposed material? Important factors were: complexity of the answer, credibility of the answer, authenticity of the answer, transparency of the answer, compliance with the requirements, quality of arguments. Topics of these problems include:

B2.1 The material concerns the exciting field of prime numbers. It serves as an excellent overview over the fascinating world of prime numbers for mathematically gifted and talented high school students. Starting with elementary facts about prime numbers, this overview touches at various places also questions that are investigated by current research. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition and reflective thinking.

B2.1.1 primenumbers.pdf

B2.2 The problem is in the field of algebra, the chapter "Equation of the second degree". Students, in solving this problem, use and apply Viète's relations and the condition that the second-degree equation admits real solutions, integers. Students use appropriate notations to facilitate calculations and abbreviated calculation formulas. The teacher explains the use of the algorithm and mathematical concepts in finding the solutions of the second-degree equation. Mathematical talented students can work independently or in groups, analysing the methods used. The teacher will find other examples with complete solutions and didactic comments. The problem provides an excellent context for the development of intuitive thinking and deductive reasoning skills.

B2.2.1 Equation of the second degree.pdf

B2.3 The problem is from the chapter "Strings of real numbers", and solving it requires special mathematical skills and the correct use of mathematical algorithms. Students talented in mathematics, will use the notions learned in the chapter "Geometric progressions" and "Inequalities". Guided by the teacher, students will prove that the set is empty, and being made up of natural numbers, admits a minimum. The teacher instructs the students that in order to prove point b) of the problem, they must apply mathematical induction and finally properties of inequalities. The teacher explains the principle of mathematical induction to demonstrate statements of type $P(n)$, any natural n . Mathematical students can work in groups, analysing and determining the hypotheses needed to obtain the conclusion.

B2.3.1 Strings of real numbers.pdf

B2.4 The problem is in the field of studying derivable functions and their properties. In solving this problem, students will make convenient notations, observing the injectivity of the function g , which is also continuous, so strictly monotonous. The teacher instructs the students how to use the theory of the monotony of derivable functions and the determination of extreme points, using Fermat's theorem. Solving this problem develops in talented students in mathematics, the ability to make connections that allow the most complete explanations regarding the properties of derivable functions. The problem provides useful concepts for math circles that include students interested in math analysis.

B2.4.1 Derivable functions.pdf

B2.5 The proposed problem is from algebra, the chapter "Exponential equations". Students notice that the solutions of the equation are 0 and 1. But then prove the uniqueness using the fact that the graph of a strictly convex function is intersected by a line at most two points. The teacher can propose to the talented students in mathematics an alternative solution invoking the inequality of Bernoulli and the monotony of the function. For the second part of the problem, the students use the injectivity of the function f . Solving the problem aims at developing the capacity to make connections that allow complete explanations regarding the study of functions. Mathematical talented students will be motivated to study and apply mathematics in a variety of contexts.

B2.5.1 Exponential equations.pdf

B2.6 Problems is mathematical analysis, in the chapter "Strings of real numbers". In solving this problem students will use the knowledge regarding geometric progressions, inequalities and will demonstrate the inequality obtained, through the method of mathematical induction. The teacher indicates to the students how to use the theoretical results in the case of the series, selecting appropriate work strategies in solving such problems. Students are provided with an excellent context for developing intuition and for the correct use of algorithms in the case of strings. Mathematical talented students can solve the problem individually, having the opportunity to compare and identify similarities and differences that occur in problems with real number strings.

B2.6.1 Strings of real numbers2.pdf

B2.7 The problem is vector geometry. In solving this problem, students are put in the situation to apply the theoretical knowledge regarding vectors, operations with vectors, non-linear vectors. The teacher guides the talented students in mathematics to determine the center of gravity of the triangle ABC , specifying its vector property. Together with the students, the teacher determines the hypotheses needed to obtain the conclusion. students who can also make generalizations.

B2.7.1 Vectors.pdf

B2.8 The problem is in the field of algebra, the chapter "Matrices and determinants". The advantages of solving such problems are multiple: a higher level of thinking is formed; a strong motivation appears; interpersonal and group understanding is promoted. Students, in solving the problem, apply properties and calculation rules of the determinants, write correctly in formal language the solution or strategies for solving such a problem. The teacher explains the use of algorithms and mathematical concepts in solving problems with matrices, matrix traces and determinants. Students can find problems challenging and they can work independently or in groups. It provides an excellent context for the development of intuitive thinking and deductive reasoning skills. The problem provides useful concepts for mathematics circles that include students interested in algebra.

B2.8.1 Matrix and determinants.pdf

B2.9 The problem is in the field of the set of complex numbers, properties and operations with complex numbers. Students are familiar with the notion of complex number, which is not the result of a measurement, as in the case of a real number, but although it has an abstract character, has multiple practical applications in fields such as electrical engineering, telecommunications, mechanics, astronomy, atomic physics, etc. are guided to choose solving strategies in order to optimize calculations with complex numbers. The teacher guides talented students in mathematics to determine analogies between the properties of operations with complex numbers written in various forms and their use in solving problems.

B2.9.1 Complex Numbers.pdf

B2.10 The problem concerns the activity in the field of studying the functions and their properties. In solving this problem, students are put in a position to apply the definition of even and odd functions. The purpose is to form the habit of resorting to different types of representations of the data contained in mathematical statements as a starting point for intuition and justification of ideas or methods of solving. The teacher indicates to the students how to use the results and the solving methods in order to select appropriate work strategies in solving problems with different types of functions. The aim is to develop interest and motivation for the study and application of mathematics in various contexts, especially for students talented in mathematics.

B2.10.1 Functions seem; odd functions.pdf

B2.11 The MATHEU Volume 2 is a set of mathematics problems for gifted students over the age of 14 and they are designed in a latter form of difficulty. The problems are the outputs of the EU funded project MATHEU. The design is made so the higher the level reached the more

talented the student is. At the same time the climbing of the latter serves as a motivation to students to reach higher level. In this manual the reader can find problem covering the following Mathematics topics: Dirichlet Principle, Mathematical Games, Geometry in the Plane, Mathematical Induction, Inequalities, Linear Algebra, Number Theory, Transformation Methods, Complex number in Geometry, Sequences.

B2.11.1 MATHEU-Volume3.pdf

B2.12 This problem falls in the intersection of logical thinking and computational mathematics. This is a good example in helping students develop problem solving skills, apply structured and logical thinking as well as deductive reasoning. Personal attributes such as confidence, persistence and organization in approaching a problem are also developed. This problem is a great chance for students to apply their prior knowledge in several notions of mathematics in order to reach a solution.

B2.12.1 Problem 1 with solution.pdf

B2.13 This problem deals with inductive and deductive reasoning and critical thinking. It involves analytical thinking and expects the students to construct logical arguments, expose illogical arguments and re-evaluate their resulting argument. It promotes rational and logical thinking. This problem is also a good opportunity for students to be asked to work not only individually but in groups as well in order to improve their skills in communicating a mathematical solution as well as commenting and reviewing on the reasoning of their peers.

B2.13.1 Problem 2 with solution.pdf

B2.14 This problem deals with mathematical reasoning, analytical thinking and optimization. It promotes the students' ability to combine pieces of information to form general rules or conclusions. This problem is good for differentiating the students' ability to choose the right mathematical method to approach the problem, as some approaches might be time consuming. This problem is ideal in promoting heuristic strategies in problem solving. It also promotes the skills of communicating a solution which is often a difficulty students are met with; how to explain their conclusion with a mathematical idea.

B2.14.1 Problem 3 with solution.pdf

B2.15 The material concerns the exciting field of prime numbers. It serves as an excellent overview over the fascinating world of prime numbers for mathematically gifted and talented high school students. Starting with elementary facts about prime numbers, this overview

touches at various places also questions that are investigated by current research. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition and reflective thinking.

B2.15.1 Prime-numbers.pdf

B2.16 The material deals with a problem of computing optimal timetables in public transit networks. It provides a good opportunity to get insights in the field of mathematical optimization. The students will develop or enrich their skills of logical conclusions. Furthermore, they can train their problem-solving abilities by processing given statements of conditions of a fictive timetable.

B2.16.1 Rudolph's timetable.pdf

B2.17 The material contains a problem of an optimal auction for multiple goods. It provides insights in the field of tropical geometry, an application of the probability theory. The students will enrich their skills of the computing of probabilities, prices and expected revenues. In addition, the teachers can use it as an application-oriented example in the field of business mathematics.

B2.17.1 Everything must go!.pdf

B2.18 The material concerns the field of cryptography. It serves as an excellent introduction to the field of asymmetric cryptosystems and addresses high school students with interest in number theory; in particular, a detailed description of the RSA public-key cryptosystem is given. The prerequisites are basic facts from elementary number theory. The material provides an excellent context for developing intuition, reflective thinking, and deductive reasoning abilities. The teaching and learning material are particularly useful for mathematically gifted and talented high school students.

B2.18.1 RSA encryption.pdf

B2.19 The materials present teachers with a number of proven possibilities for effective learning of mathematics, instead of teaching how to play algorithms for standard exam tasks. All proposals are checked in practice. It is certainly worth trusting this guide and try to instil the ideas presented in it on your own foundations, adapting them to your needs. Mathematical interests and talents of students are rarely communicated to teachers. In most cases they must earn it solidly thanks to enthusiasm, commitment and ingenuity and professionalism. This guide will be helpful because it shows you how to work with ordinary students, arousing their interest

and motivation to become extraordinary students. The authors try to answer questions on how to educate thinking, creative and inventive students, how to instil mathematical passions in them and how to develop interests, and finally - how to educate key mathematical skills: logical thinking, precise argumentation and use algebraic techniques and perception of geometric relationships.

B2.19.1.jak_pracowac_z_uczniem_zdolnym_poradnik_nauczyciela_matematyk.pdf

B2.20 The materials present the concept of teaching the probability theory based on the concept of a stochastic graph. Probability theory today has more and more applications in various areas of human life. The textbook gives a different perspective on this section of mathematics. Basic probabilistic concepts are introduced in the context of generalizations of Penney-Ante games. The basic tool in solving various problems is the stochastic graph. Thanks to this, the student gains an interesting, intuitive and very effective conceptual apparatus.

B2.20.1 Tlusty, Krech – MONOGRAFIA.pdf.

Chapter 3: Projects and Applications

This section includes older completed projects of different types and current running projects. Like the previous two sections, this one was created as a result of cooperation of Partners in the scope of searching for projects and applications helpful in working with a talented/gifted student. The found materials are to serve as a model for implementation and also to show recent project results and case studies of practitioners. The spectrum of interest includes such materials as: data base with tasks and exercises (without solutions), data base with tasks and exercises (with solutions), tutorial given by a teacher, tutorial given by a student, forum moderated by a specialist, forum moderated by a student, interactive media, interactive media, individual games, cooperative or collaborative games, project problems, competitions. To make the review of materials easier and more useful for the reader, we have divided it into subsections: Projects/Investigation and Applications related to other sciences (STEAME) and real life, Projects/Investigation and applications related to gaming, projects/Investigation and applications related to industry/business world/thematic/research institutes/authorities/organizations relating to real life issues, Projects and applications related to cooperation with universities.

B3.1 Projects/Investigation and Applications related to other sciences (STEAME) and real life

B3.1.1 GOLDEN RATIO - NUMBER AND UNIVERSE is a complex and elaborate project, which involved students aged between 16 and 19 from Italy and Romania. The project aimed to arouse students' interest and curiosity in mathematics, proving that it has always been an essential part of human development and culture. The students were tasked with searching for information about the gold ratio and the Fibonacci series; notation of links and definitions of new notions; group activities in order to clarify the theoretical concepts; developing applications in collaboration with teammates; evaluating and, possibly, correcting some applications of the other colleagues; participating in discussions with coordinating teachers.

B3.1.1.1 A GOLDEN RATIO - NUMBER AND UNIVERSE - EN.pdf

B3.1.2 The ratio of the Gifted European Mathematicians (G.E.M.) project was born from the recognition of the need for many European countries for highly qualified youngsters in mathematics, science and technology, that are available to mobility, with high language skills and able to propose innovative solutions for the social and economic growth. The context of the school partners in G.E.M. arises from the needs identified on the target group (students 14-17 years old) and identifies two topics with similar needs: Math Literacy, English scientific skills.

B3.1.2.1 Gifted European Mathematicians (G.E.M.).pdf

B3.1.3 The “Stiftung Planetarium Berlin” unites the astronomical institutions of Berlin. The Archenhold-Sternwarte and the Wilhelm-Foerster-Sternwarte are among the most traditional public observatories in Germany, while the Planetarium am Insulaner and the Zeiss-Großplanetarium, the most modern science theater in Europe, offer relevant and innovative forms of education. As a joint institution, they are an essential part of Berlin’s educational and cultural offerings. In addition to astronomy, the range of topics will be extended to other sciences. In the near future, materials for mathematics workshops accompanying the astronomical events of the planetariums shall be offered in cooperation with the Humboldt-Universität zu Berlin.

B3.1.3.1 Planetarium.pdf

B3.1.4. Zero Robotics is a space-based programming competition for middle and high school students. Massachusetts Institute of Technology (MIT) runs the program in collaboration with NASA and CASIS. Each year features a new “game” based on a real-life space exploration challenge selected by MIT and NASA. Competing student teams must program their robots to manoeuvre with precision and strategy, protecting their own robot while trying to thwart their opponents’ efforts. The competition uses soccer-ball-sized robots, that float inside the International Space Station and manoeuvre with small puffs of compressed gas.

B3.1.4.1 Zero Robotics.pdf

B3.1.5 F1 in Schools is the only global multi-disciplinary challenge in which teams of students aged 9 to 19 deploy CAD/CAM software to collaborate, design, analyse, manufacture, test, and then race miniature compressed air powered F1 cars. Providing opportunities to learn Science, Technology, Engineering and Maths (STEM) related subjects in an exciting way. The challenge inspires students to use IT to learn about physics, aerodynamics, design, manufacture, branding, graphics, sponsorship, marketing, leadership/teamwork, media skills and financial strategy, and apply them in a practical, imaginative, competitive and exciting way. During the development of their cars teams need to follow a handbook of regulations followed in the real F1.

B3.1.5.1 f1inschools.pdf

B3.1.6 NASA Space App Challenge. This Challenge takes place in Nicosia Cyprus every year. Participant teams had to prepare and present a project entitle: How would you do about colonizing the moon, which included defining the contributing members of the base, designing a list of requirements for countries to contribute in such a project, decide how many people from each country or state should participate, structure a leadership and governance for the colony(ies), design a social structure etc. The teams had one weekend to think about their project

and ten minutes to present it before the judges, two from the CSEO and one from NASA. NASA offers the winning team the unique opportunity to visit the Marshall Space Flight Centre where astronauts are trained, while three more teams will visit the Rocket and Space Centre. All teams will be given the chance to train like astronauts for one week. Every year students are given a new topic to work on.

B3.1.6.1 Spaceappschallenge.pdf

B3.2 Projects/Investigation and applications related to gaming

B3.2.1 Math in Wonderl@nd: with only two partner schools, from Italy and Romania, this European Mathematics and Science Prize-winning project started with the mathematical allusions in "Alice in Wonderland" to show students that math can be "friendly", full of mystery and imagination, to help them discover and develop their ICT, social, linguistic skills and creativity through collaborative activities and active learning. Within the project mathematical games were developed Because Mathematics is such a serious subject, it has become a pleasant and friendly discipline by creating games developed in the Mathematical Games section of the online magazine, as well as on the blog. The project aims to enhance the students' motivation by making Mathematics a learner-friendly subject, to help them discover and develop their creative, social, language and ICT skills, by means of collaborative work and active learning.

B3.2.1.1 Math in Wonderl@nd CCDTR_EN.pdf

B3.2.2 Mathe im Advent – Making Math fun and relevant. The German Mathematical Society (DMV) runs this online Advent Calendar each year in Christmas time for two age groups: From grades 4 to 6 and 7 to 9, respectively. With interesting mathematical problems and concepts, which are usually not taught at schools, it reveals to more than 100,000 school kids the often-surprising message: Math can be fun and useful!

B3.2.2.1 Mathe-im-Advent.pdf

B3.2.3 The MATH+ advent calendar offers fascinating insights into recent application-oriented mathematical research and into the everyday professional life of mathematicians. The 24 challenging puzzles are aimed at bright students from grade 10-13, university students, teachers and everyone interested. Therefore, the calendar is intended to challenge and support (not only) gifted pupils in mathematics. All participants can discover the power and beauty of mathematics and the best ones can win prizes.

B3.2.3.1 MATH+ advent calendar.pdf

B3.2.4 Math Game Day – Students in teams according to their year group, compete online in real time with students from other schools all over Cyprus and abroad. They are given a series of short calculations (+ - * /) to do in a specific amount of time, each game consists of 3 rounds for each team/year group and it lasts about 20 minutes with breaks.

B3.2.4.1 thalescyprus.pdf

B3.2.5 The idea of a mathematics Hackathon organized either internally or internationally among schools is also a good opportunity for gifted students in mathematics to further expand their mathematical skills but also to be given the opportunity to work as a team.

B3.2.5.1 maths-hackathon.pdf

B3.3 Projects/Investigation and applications related to industry/business world/thematic/research institutes/authorities/organizations relating to real life issues

B3.3.1 The European Light Infrastructure - Nuclear Physics (ELI-NP) project is a project that aims to develop the most powerful laser in the world and create a pan-European research center in Romania, at Magurele. This is the first of its kind in Europe, the results of the developed technology can have applications in fields such as medicine, nuclear physics, electronics, information technology, pharmaceuticals, waste recycling technology, etc. Extreme Light Infrastructures (ELI) project aims to establish European laser research facilities in Romania, Hungary and Czech Republic for the investigation of high-intensity light-matter interactions and their potential applications in science, industry and medicine. Thales developed with ELI-NP (Nuclear Physics) in Romania an unique laser system made of two beam lines each delivering, a peak power of 10 PW at 1 shot/min, with intermediate outputs at 1 PW, 1 Hz and 100 TW, 10 Hz.

B3.3.1.1 European Light Infrastructure project_EN.pdf

B3.3.2 Jugend forscht encourages and supports talented achievers in the areas of science, technology, engineering, and mathematics (STEM). It seeks to inspire young people to become involved for the long term, and, after the contest, to help them with their careers. "Jugend forscht" is the largest public-private partnership of its kind in Germany. There are about 250 partners, most of them from the business community. They organize the contests, donate the prizes and lend their support to other activities. The Federal Ministry for Education and Research (BMBWF) defrays the running expenses of the Jugend forscht head office.

B3.3.2.1 Jugend-forscht.pdf

B3.3.3 Research and business expeditions are expeditions for a team of 15 - 40 German-speaking students to interesting regions (e.g. Iceland, North Cape, Azores ...) in order to experience the research process for themselves. They explore fauna, flora, waters, climate, human-culture-language, infrastructure and economy. This means that all parts of the expedition are

experienced during the course of the expedition, from the setting of objectives to the planning of experiments and from data collection to the complete documentation. The young researchers have to carry out experiments, surveys and data collection, arrange, evaluate, and document them and finally present their results.

B3.3.3.1 Research and business expeditions.pdf

B3.3.4 London Science Forum. The London International Youth Science Forum (LIYSF) is a two week residential student event held at Imperial College London & The Royal Geographical Society, with lecture demonstrations from leading scientists, visits to world class laboratories and universities combined with cultural interaction, with 500 students aged 16- 21 years old from 70 countries.

B3.3.4.1 liysf.pdf

B3.4 Projects and applications related to cooperation with universities

B3.4.1 One was an Approach to Mathematically Gifted by Dr Boštjan Kuzman Through examples, he presented different situations that can arise while working with mathematically gifted students and can be a cause of embarrassment for the teacher as well as the students. Apart from the mathematical knowledge and appropriate motivation, the teacher who works with mathematically gifted must also provide appropriate and socially secure environment. He was determined to solve a couple of mathematical challenges along the way. Another one was Scratch & Bebras Puzzles by Dr Irena Nančovska Šerbec & Špela Cerar In recent years, computational thinking has become one of the big issues in education worldwide. Various learning activities and competitions were created for development of computational thinking skills. In this spirit, several summer schools were conducted for primary school pupils using ScratchJr and Scratch.They presented some of the ideas that can be used for developing games and stories with gifted kids. Bebras competition is run in Slovenia from 2011. Lots of different tasks and puzzles were created during this time. They presented some ideas how to use the puzzles for learning of fundamental concepts of computer science. Problem questions:

1) What programming activities in Scratch are interesting for gifted students? 2) Are tasks which teachers find difficult also challenging for gifted students? 3) What makes a task interesting for gifted students?

B3.4.1.1 Summer School of Science and Technology from Măgurele_CCDTR_EN.pdf

B3.4.2 Summer School of Science and Technology from Măgurele aims to enhance the scientific expertise of the education and research platform at Magurele and the University of Bucharest making it accessible to high school students regardless of their profile and specialization and teachers who teach STEM subjects (science, technology, engineering, mathematics) in secondary and secondary education. The school proposes a complex complementary program based on the

premise that STEM education through its purposes is a vector of impact on increasing competitiveness and social cohesion, in the context of sustainable development of Romania with significant influences on increasing the welfare of both learners and the community in which they live. The trainers and mentors at the summer school are researchers, teachers from pre-university and university education in the fields of STEM, education sciences but also from other specializations, the approaches being of integrated type.

B3.4.2.1 EGIFT Project-European Gifted Education Training.pdf

B3.4.3 The Mathematische Schülergesellschaft “Leonhard Euler” (MSG) is an extra-curricular, free institution for the promotion of mathematically interested and talented secondary school students. It is organized by the Institute of Mathematics of the Humboldt-Universität zu Berlin in cooperation with other Berlin universities. In about 30 weekly courses (“circles”) mathematically interested and talented pupils from grades 5 to 12 are given exciting insights into various areas of mathematics beyond the school curriculum. The focus is on problem-oriented work, scientific methods, and training for mathematical competitions.

B3.4.3.1 MSG.pdf

B3.4.4 The Summer Schools “Lust auf Mathematik” take place every year and last one week. During this week interested and talented students from grade 11 work in small groups on challenging mathematical problems. They are instructed by professors and scientists of the Humboldt-Universität zu Berlin and other universities of Germany. At the end of the week the students present their results by a talk and document their work with a 10-page report.

B3.4.4.1 Summer-schools.pdf

B3.4.5 The „Berliner Netzwerk mathematisch-naturwissenschaftlich profilierter Schulen“ is a network of five grammar schools, the Humboldt-Universität zu Berlin and the state government of Berlin. It was founded in 2001. This network is promoting students who are talented in mathematics and the natural sciences. This is achieved by an enriched curriculum in mathematics and additional semester hours starting in grade five. Students learn about mathematical structures, argumentation and proofs. They have the opportunity to obtain credit points for the first semester of university education in mathematics. The grammar schools have approximately 1800 students in classes with a focus on mathematics and the natural sciences.

B3.4.5.1 Berliner Netzwerk mathematisch-naturwissenschaftlich profilierter Schulen.pdf

B3.4.6 The University of Cyprus Department of Mathematics and Statistics organises a workshop for high school students who love Mathematics. This year’s workshop was entitled: ‘Why Math’. Students had the opportunity to listen to interesting speeches from experts and university professors on what they can do with Mathematics, what skills they can develop and what careers they can follow with a degree in Mathematics.

B3.4.6.1 WhyMath.pdf

Chapter 4: Competitions/Challenges and Communication activities

B4.1 FutureOlympics is a project realized in partnership between Intuitext and the Romanian Society of Mathematical Sciences. Through this project, the Olympics in mathematics training are supported. Together with the professors from the Romanian Mathematical Sciences Society, keeping the tradition of the Mathematical Gazette, Intuitext brought online the performance in mathematics! Every year, the competition on the FutureOlympics website has several stages (students solve problems, send solutions and receive points depending on performance) and is completed with a camp in Câmpulung, where the best students participate for free and experience mathematics otherwise. The competition is part of the Mathematics Education Otherwise Program, a program of the Romanian American Foundation implemented in partnership with the Romanian Mathematical Sciences Society.

B4.1.1 FutureOlympics.pdf.

B4.2 The Speed Kangaroo Competition is a group competition based on the world-famous Kangaroo competition. Students solve Kangaroo tasks in groups of up to six participants as quickly as possible. The fastest group sets the processing time – but be careful, for every wrong answer the other groups get additional time.

B4.2.1 Speed-Kangaroo-Competition.pdf

B4.3 The Berlin Day of Mathematics is a mathematical event that takes place at a different university in Berlin every year since 1995. It is aimed at students and teachers who are interested and gifted in mathematics. The aim of this event is to maintain and develop enthusiasm for this science with exciting tasks, inspiring lectures and interesting workshops about the fascinating world of mathematics. Furthermore, team competitions will be held on this day so that social and mathematical skills are supported by solving challenging tasks.

B4.3.1 Berlin Day of Mathematics.pdf

B4.4 The Mathematics Olympiad in Germany is a competition for (gifted) students in mathematics from grades 3 to 13. In this competition, students have the opportunity to prove and develop their mathematical skills. Challenging tasks promote logical thinking, reasoning abilities, the ability to combine and the creative use of mathematical methods. The competition extends over several rounds - those who get to the federal level can hope to win Olympic gold. The best participants will be delegated as the German team to the International Mathematical Olympiad.

B4.4.1 Mathematics Olympiad in Germany.pdf

B4.5 Regional Mathematics Competition – students have two hours and are expected to work on about 4 questions that require a structured answer. Also, the Mathematics Olympiad organized by the Cyprus Mathematical Society is a good competition for gifted students in Mathematics to compete, show their abilities in Mathematics and gain experiences for future competitions regarding university entries and university life.

B4.5.1 competition-cms#competitions.pdf.

B4.6 Math Relay Competition – Students are in teams according to their year group. We have three teams (Years 7 - 9) each consisting of three students. Then the Year 7s get 15 minutes to answer one question that requires a structured answer. When they finish Year 8s have 30 minutes to answer their own question and Year 9s have 45 minutes for their own. In case a group has a few minutes that they did not use the next team can use this time, and that is why it is a math relay.

B4.6.1 mathematics-Relay.pdf.

B4.7 Math Factor and Science Factor competitions within the framework of Euromath & Euroscience Student Conference: Students need to communicate/present a mathematical or scientific concept in a funny and simple way so that it is comprehensible to even a non-mathematical or scientific oriented audience. Also, Math Theatre and Science Theatre Competitions are an excellent medium for students to communicate science and mathematics.

B4.7.1 euromath.pdf

B4.8 Krakow Youth Society of Friends of Sciences and Arts at the Youth Center dr. H. Jordana and the Cracow Branch of the Polish Mathematical Society invite you to the MAŁOPOLSKA MATHEMATICAL COMPETITION ending with the Małopolska Mathematical Session. We encourage students from secondary and primary schools to participate in the event. Every year, the competition gathers enthusiasts of mathematics from the entire Małopolska province. Some do their work alone, others choose to work in a group. The event provides for two forms of work implementation: individual and team (a team cannot have more than 3 people). Below are sample topics for papers: Presentation of the problem which is dealt with by young people during mathematics club classes. You can present methods (also ineffective) that tried to solve the problem. Presentation (in a way that is understood by peers) of theories or concepts from extracurricular mathematics. Solution to an interesting task. You can present different methods to solve the same task. Unusual, unknown evidence of known mathematical facts Development or implementation of scientific support or measures to popularize mathematics among peers. The awarded works are published on the website www.timediawo.edu.pl, which can help in choosing the topic of work.

B4.8.1 małopolski_konkurs_prac_matematycznych.pdf

Chapter 5: Videos and Interactive Media related to giftedness

B5.1 A solution that they propose for this situation is the online mathematics course, based on 40 video-learning units for different topics (ex: Circle, Rectangle, Sequences, Factors and multiples, etc.). This course can be freely accessed by anyone who is interested in mathematics – students, teachers, parents. It can be accessed by any device, from your class or from your home. You can access the online mathematics course from the e-learning platform Menu Cursuri/Mathematics Open Online Course. To access this course, you must create a username and password. After these 3 steps, you gain access to 40 video tutorials about maths and more than 200 exercises. Everything is free! The 40 video tutorials, available in both English and Romanian Language, can also be accessed from the YouTube Channel dedicated.

B5.1.1 mathematics-open-online-course.pdf

B5.2 Gifted and talented children: signs and identification. In this video, parents of gifted children talk about noticing early signs like a big vocabulary, advanced use of language or musical talent. They talk about having mixed feelings and working out what it means for their child's future. An educational psychologist says it can help to get a formal assessment when a gifted child is approaching school age, to help the school plan for your child's educational needs.

B5.2.1 Gifted and talented children- signs and identification.pdf

B5.3 Gifted, creative and highly sensitive children. Being “gifted” often feels far from a good thing, when you're the one living it. And yet the myth persists that “gifted” is an elitist and privileged label in education. That “those kids” will be fine without any extra programs or supports. After all, what's the worst that can happen – they get bored? In reality, many of our most vulnerable learners are those “gifted” kids who know they just don't “fit it.” In our rush to identify and label children for purposes of funding and organizing our educational services most efficiently, we're missing the ground level, actionable and everyday needs of an entire grouping of needy kids. Our most gifted, sensitive and creative students are our canaries in the coal mine; they are more sensitive to the world around them and the reactions we see in them can teach us what our education system needs to be, believe, include and stop. As is often the case, what is necessary for some is beneficial to all.

B5.3.1 Gifted, creative and highly sensitive children.pdf.

B5.4 The Gifted Adult. Lynn speaks to her experiences as a gifted person and describes the unique challenges faced by people with high IQ. The gifted's propensity towards perfectionism and their attraction to extremes should be carefully managed to ensure their talents have a life-enhancing effect.

B5.4.1 The Gifted Adult.pdf.

B5.5 The videos deal with the derivation of the Euclidean algorithm to determine the greatest common divisor. They are based on the fact that a natural number can be represented uniquely by division with remainder. Then the algorithm is derived successively and applied interactively by tasks of exercises. It serves to promote algorithmic and number- theoretical thinking and requires the handling of numbers, operations, arithmetic laws and divisibility. This sequence is aimed at students up to the age of 14 years. The material and the videos are prepared in German language.

B5.5.1 Euclidean Algorithm.pdf

B5.5.2 Euclidean Algorithm2-video.pdf

B5.5.3 Euclidean Algorithm3-video.pdf

B5.5.4 Euclidean Algorithm4.video.pdf

Part C: The Mathematics meets Industry in School model –Guidelines for implementation of a Mathematics Industry Day (MID) (EN)

Chapter 1: Guidelines for a MID-day

You first have to identify the gifted students and then convince an industry that their lateral thinking capacities might be worth a try. At the same time, you have to motivate academics and teachers to back you up and then you have to plan and design a MID-day (a Mathematics meets Industry Day). This chapter will provide the guidelines for planning and conducting a MID-day event.

1.1: Definition and Glossary

Partner: In these guidelines, when we refer to partners, we mean an industry ready to invest a day or two talking with academics, teachers and students in order to define and address an industrial problem.

Industrial problem: A suitable issue brought forward by the industrial partner, that is understandable by students, teachers and academics. It has to be adapted on both sides, address some useful issues on the side of the industry and be challenging, yet feasible on the side of students. The state of the problem is the result of a common brainstorming between the industrial partner, the academics and the teachers, prior to the MID-Day. A list of possible problems is provided in section 1.4 below.

MID-Day: An actual Math and Industry Day, when students tackle a problem set up by an industrial partner and adapted by the academics and teachers. It might be more an ideation, the refinement of an idea, than actually the full solution of a problem.

1.2: Methodology, Requirements and guidelines for schoolteachers (the teacher as a mentor and co-creator)

The responsibilities of the teachers are to select and prep-up students in a way that will make them fit for a fruitful MID-Day. On the MID-Day, teachers should facilitate students' work. This means that first you have to select students, whether using a written test or based on preselection by parents' association, grades at school, Olympiad or math clubs selection... Also you have to obtain formal agreements from parents and schools for the participation of everyone.

After your first discussion with the industrial partner and the academics, you must decide on the type of MID-day you wish to conduct. You will find below a few proposals. You will have to set up a scratch course based on a lesson plan that we provide, to level up the students to the adequate degree. This implies as well adjusting roles according to levels: some more advanced or mature students can act as coaches or tutors to younger ones, as *guide on the side* rather than *sage on the stage*.

Facilitation during the MID-Day involves propping up intimidated students, supporting self-confidence, reassuring them that their knowledge is not insufficient to start thinking about the problem and easing communication, making sure that no one feels insecure or imposes one's view, that every voice is heard, that every student feels needed and valued in a way or another. Division of the problem in sub-tasks has to be planned ahead for this purpose.

As a last step you should report on how things went, observing regulation and self-regulation, how each student reacted, what triggers or withers his/her motivation, how he/she belonged, what was proposed and created.

After the MID-Day, you should pay special attention on how the material raised during the MID-Day can be further used in everyday math class, relating it with institutional knowledge. Further look for opportunities for students to share their work with others in fairs, competitions, forums, clubs or setting up an exhibition in your school. This will help recruiting students for the year to come. Do not forget to mention that participation in a MID-Day can be valued by students in their CV when applying for grants. This event will spring and nurture their professional network!

1.3: Methodology, Requirements and guidelines for universities supporting MID days

University partners should build the team of students that will work on the industrial problem. That means working with students but as well training students in managing work as a team. In collaboration with the industrial partners and teachers, they have to shape the problem in a way that it can be worked upon by students.

Your main responsibility is to set up, organise and manage the MID-Day and the preparatory meetings before that. A typical interaction with teachers and students will have three phases:

In a **1st meeting**: You will meet in an informal way students and their teachers, motivate them, build a team around original, creative and fun math material. You can reach out for participants in math club, science fair, math day, math competitions, inter-schools' meetings and so on.

In a **2nd meeting**, you will level up selected students into building the group as a team. That means that you will teach them the tools to work as a team (see below).

The **3rd meeting** will consist in the MID-Day *per se*. You will have to select facilities convenient and large enough for the meeting, whether at the university or at the industrial partner's site,

have coffee breaks and lunch at hand and so on. A hybrid version, with online teleconferences might be in order if no possible physical meeting is feasible.

You will have to facilitate communication and team's work, asking the right questions so that everybody feels adequate and productive. Be careful not to look for answers by yourself without relying on students. Make sure students are relevant and the focus of the MID-Day. You are not here as a teacher that teaches students, they should be seen as trainee fellow researchers.⁸

Basic tools for working as a team:

- Make divergences clear. While keeping a secure environment, reach a fruitful debate: do not vote, do not compromise, do not select at random, avoid averaging and bargaining but strive for unanimity.
- Listen and involve: Do not let anyone impose their standpoint but argue, do not dismiss but propose rebuttals.
- Do not personalize debates, do not take things personally. Look for solutions.
- Accept solutions only if it is understood and shared by everyone.
- Giving a light training in these themes, both for students and teachers, increases the probability to get original solutions and a productive team.

1.4: Proposed Examples of MID days

- Discuss with your industrial partner. Optimization problems can be found in many businesses: Whether the industrial partner is cutting glass, numerically controlling robots, managing a storage house or arranging a supply chain, they surely have a basic open optimization problem at hand that needs to be clarified if not solved.
- A **Statistical Research Survey** will interest your industrial partner, they might want to know better the needs of their clients or employees, and a market or human resources research project is relatively easy to set up, fun and creative to design and can lead to unexpected results when seen from an unusual point of view: *"how to transform this bleak open-space in the covid context?"*. A lesson plan in statistics and survey methodology is in order before actually getting down to business.
- A **Scientific Trail** on the industrial site is a great way for the industry to promote their work during class visits, and for the students to become problem posers and look at their surroundings with a scientific eye: *"how large is this tank? how long will it take to fill it up?"* The lesson plan should concentrate on modelling and measurement methodology,

⁸ Hall, Jay (1971) Decisions, Decisions, Decisions, Psychology Today, November, 51-54, 84-88.

discovering for example that your own body is a pretty precise tool for everyday quantities estimations.

- If you do not have direct contact with an industrial partner, don't forget the **educational industry**, and have the students set up pedagogical games for you: let them invent new ways to teach by gamifying some of your content. Learn from them! The lesson plan should teach neural basis of cognition, gamification, determining aims and sub-aims, defining challenges and levels, ways to collaborate, helping the students understand their own cognitive process better. And you never understand a concept better than having to explain it, especially to a computer!

Once the main theme is set, you can plan the lesson path to get the students from where they stand to the required competencies regarding the tasks at hand. That is: you have to conduct the lesson in an introduction to the project. This first day at school should be relaxed and joyful, students working in teams and getting to know one another.

Once the problem is refined with the partner, you should send it to students in advance with a possibility to react, interact, with clarifying questions.

During the big day of the MID-Day, students visit the industrial site or at least discuss with representatives of the industry and work on their project together during the morning.

Here is a possible schedule for the MID-Day:

- 9h-9:30: Welcome, small talk ice breaking
- 9:30-10:30: Team building with a group activity like « lost on the moon » or similar. <https://netmind.net/play-the-moon-landing-exercise/>
- 10:30-10:45 Break
- 10:45-11:45 Discussion in sub-groups, regarding the problem of the MID-Day, comprehension of the issue, first tracks of thoughts...
- 11:45-12:15 First synthesis on possible strategies addressing the issues, setting crews on specific tasks.
- 12:15-13:45 Lunch break
- 13:45-15:45 Work in sub-groups on specific strategies and tasks.
- 15:45-16 Break
- 16-16:30 Preparation of the synthesis and its presentation [The students]
- 16:30-17:15 Presentation in front of the industrial partners, discussion

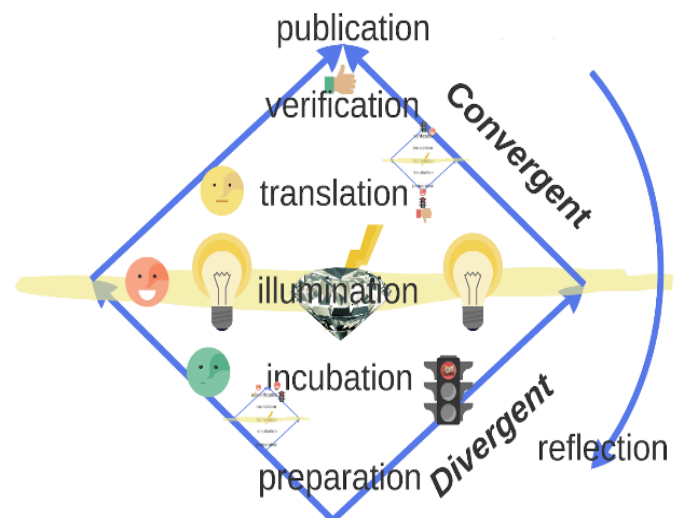
1.5: Evaluation results, Dos and Don'ts

During the MID-Day:

Beware that every student should belong to a small group, at most 8 students, which is to be formed in the preparatory activities, for example with older students supervising the group as a coach, making everybody feel needed and belonging. One has to pay a special attention to insecure students by asking suitable questions, highlighting and developing the special abilities of each student, increasing self-confidence. The industrial problem is thus analyzed with every perspective in mind, so that it is possible to divide partial tasks among all team members, fostering the collaboration of all team members.

The evaluation should focus on these aspects: that every student felt taken into account as an individual thinker and creator. Of course, not all ideas can lead to a successful product, maybe no production will result from the MID-Day, and it should not be felt as a failure, but the potentialities and the creativity of ideas and productions should be put forward. If you need numbers, or want to assign individual trophies, you can objectify the evaluation of the creativity of the process of ideation that took place along the four dimensions described by Guilford: Fluidity, Flexibility, Elaboration and Originality:

- Fluidity is the capacity to produce a lot, to work out many examples.
- Flexibility is the ability to change a strategy when you are stuck, serendipity and adaptation are clearly the keywords here.
- Elaboration digs in depth a question, looks for the fine-grained details, is exhaustive in its search.
- Originality is the type of creativity you recognize when you see it: you have never seen anything like this, it is new and provocative.



Since we are talking about ideation, be aware that we are not in the students' head, to be fully creative, an idea has to go through the whole cycle of creativity with the two moments of divergent and convergent phases: from preparation to incubation, then illumination (the Aha moment), but this idea has to get out of the student's head and translated into words, sketches and so on, in order to be verified, and then published, proposed to colleagues to see what they think about it, whether they validate it. And in the end this idea can be reflected upon and integrated into a greater cycle of ideas in order to address the problem at stake.

One should not mistake creativity with only divergent thinking: a phase of brainstorming is ok, but at some point, it has to depart from daydreaming and converge to an actual workable solution, and there is a need for as much creativity in this convergent phase. Likewise, the Aha moment of illumination is not to be overvalued, poetry and industry can mix only to a point, what seems like an original idea has to translate to actual schemes. These subtle points have to be taken into consideration by the team in order to foster a productive mindset among students.

After the MID-Day:

All results will not show straight away during the MID-Day. Students will come back home with lots of observations, ideas and plans. In the last phase, this work can be reworked and presented again to the industrial partner. It is important to organize a way for them to exchange with others. This can be further used, for school fairs, forums, schools twinning, integration weeks, competitions and so on in order to show what has been done.

1.6: Why Industry should get involved, benefits for industry

This point of the interest of the industrial partner has to be cleared beforehand but it is good to revisit it at the end of the MID-Day and go through the checklist: Have all parties benefited from the MID-Day? Has lateral thinking really unblocked a creativity situation or simply has the industry been able to show its business to outsiders. The benefits really are different depending on the type of MID-Day that was conducted. To go through the generic examples, we gave:

- Has the Statistical Research Survey been able to make evident elements that went unseen before, or was it simply a way to communicate, whether internally or with potential customers? Allowing customers or better employees to express their views, to see that they are taken into account is already something that can be valued.
- Is the Scientific Trail fun and informative? Can it be used with potential customers, with classes? Does it open the eyes of participants on the challenges that the industry addresses? In the case of a touristic visit of a site, does the partner discover the site with a new eye? Will it serve its purpose of scientific tourism?
- As for the educational industry, have student's setup interesting, informative and fun pedagogical games? Did they learn something out of designing the pedagogical game?
- The industry will have the opportunity to get in touch with gifted students and possibly work with them in the future.

1.7: Implementation of MID – Days in various Countries:

The MID- Day event was piloted and implemented in five European countries; Those are: Cyprus, France, Germany, Poland and Romania. The idea behind the MID – Day event is that the application of mathematics in real life is an absolute must for any curriculum nowadays and even more for the gifted pupils who can contribute to innovation that could make the human life better and at the same time promoting entrepreneurship in the school curricula. The Mathematics meets industry event is designed to offer challenging unsolved real-life industry problems to students with the aim to work either in teams or individually and use mathematics in order to propose a set of possible solutions to the given problems. Furthermore, students had the opportunity to collaborate with each other and work together towards a common goal, meet Higher education researchers as well as Industry people and for secondary school teachers to see, get ideas and get inspired on how such problems could be used as part of their regular teaching.

The preparation and implementation of the event varied in each country. Various industry problems were presented, which students were challenged to solve. Therefore, below we will analyze the preparation and implementation of each MID-day event by Country. This way examples and suggestions on the MID-day methodology can be extracted.

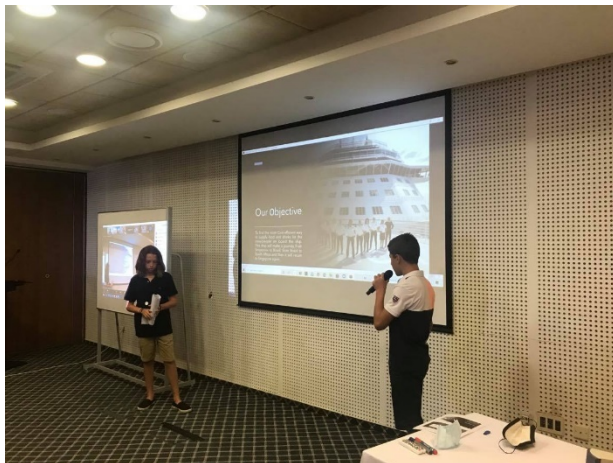
In the Case of Cyprus

Preparation actions: The preparation for this event started two months in advance when an invitation/motivation letter was sent to students that enjoy mathematics and might have been interested in such an activity. In the meantime, the INNOMATH teacher team of the Heritage Private School contacted local companies, following the organisation of a Focus Group, and asked them to join this event by providing the students with a real-life industry problem. Two companies provided a problem each for the purpose of the MID-Day. Problem A was provided by V-ships which is one of the biggest shipping companies in the world. The problem described a specific case of a tanker vessel trading between Singapore and Brazil that needs to be stocked up with provisions in order to have provisions on board at all times. In this project, the students needed to act on the benefit of the catering provider in an attempt to formulate the most cost-effective provisions replenishment plan by making sure that the agreed rates are met, the vessel will have sufficient stock on board at any time during the voyage and at the same time they had to maximize the profit for the catering provider company. Problem B as given from an architecture company and the students were asked to take the place of an architect, to understand the process and to find the best possible solution to serve a client based on his house requirements, materials to be used and available budget.

Students preparing their MID-DAY presentations: Many students who love mathematics, opted to participate in the MID-Day activity. It was an opportunity for them to think creatively and outside of the box either individually or in teams in order to find possible solutions to the given problems in an outside the classroom activity. Due to Covid19 restrictions and students participating coming from different cities, two online meetings were organized for the Heritage

schoolteachers, the participating students, the company representatives and the Mathematical Society. The online meetings took place in early and mid-July. The aim of the meetings was for students to create their teams, become familiar with the problems, get extra explanations and clarifications from schoolteachers as well as the company representatives. General guidelines for the smooth running of the MID-Day and the requirements for the student presentations were also given. After the first meeting students had some days to decide on their preferred problem and started to work on it. Problem A was more popular as it was selected by all students. Then, another online meeting took place for students and teachers to discuss the progress of the designing of the solution. Students were advised on how to organize their work and how to split the workload within the team. Students also had the chance to ask for more assistance and clarifications that could help them working through their proposed solution.

The actual MID-DAY: On Wednesday 28 of July 2021 the event took place in Agros in the presence of students, parents, teachers, higher education researchers and project partners. The students presented their possible solutions and at the end of each presentation the representative of V-ships, Mr. Loucas Chrysochos, was giving his feedback on the solutions students proposed. Also, all the other participants were free to ask questions to the students. All the solutions received from the students were different and this initiated a fruitful discussion between the participating students and the audience. At the end, the industry representative expressed his excitement for the students' hard work, excellent presentation skills and the analytical way of presenting their solution. **He mentioned that it was a great experience for him to participate in that event and meet such students who did a great job. In addition, he expressed his interest to work with them one day.**



In the case of France

The actual MID-DAY:

Students taking part in the MID-Day wear two hats, they are aware that they are not only participating but as well testing the pedagogical innovative plan. The students were set up in a creative mind set, where mathematics were viewed as a concrete tool to solve real life problems of an economic value.

The students were let to set up in teams through icebreakers, then installed the MathCityMap application and run the local math trail.

They spent the best of the day in the field, in the UNESCO heritage Renaissance quarter, listening to the stories that the professional guide was telling, taking notes, setting things up in a team, trying out ideas, letting other members of the team check the feasibility and fun of the tasks... Back at the university, the tasks were encoded in the MathCityMap system.

The main issue was to answer the needs of the tourism industry representative: students discussed the best ways to address different targets, families, classrooms, groups of friends. Getting to understand the constraints of the professional guide, infusing enough fun and eye-opening information within scientific questions is the main goal of the mathematical trail that was produced.

The second part of the MID-Day was the feedback to the industrial partner. Students had the opportunity to team up in order to set up their talk. It took place during the transnational project meeting that took place in Lyon mid-December 2021 on the UCBL campus. The day after the trail the students justified the choices that they made in front of the jury composed of the industry representative and the international partners of the InnoMath project.



In the case of Germany

Preparation actions: The preparation for the MID-day event started in 6 months in advance with a meeting with industry partners discussing potential tasks. The Focus Group meeting, which consisted of teachers, scientists, and industry partners, met and evaluated the ideas and the proposed schedule for the MID-DAY. Later on, schools with a focus on mathematics and the natural sciences were invited to take part in the event. Together with the Stiftung Planetarium Berlin, it was decided to pose the problem to the students of how changes in the seasons - climate change - can be made statistically visible and how predictions can be made. In particular, the models used were to be evaluated and reflected upon.

Students preparing their MID-DAY presentations: 14 classes (420 students) of schools with a focus on mathematics and the natural sciences participated in the MID-DAY activity. Due to Covid19 restrictions the students had to meet in their schools and where not able to meet students from other schools for the preparation of the presentation. The preparations took place in September and the students were given material to become familiar with the problems, which was prepared by the researchers and the industry partners. The teachers supported the students and supervised them. General guidelines for the smooth running of the MID-DAY and the requirements for the student presentations were also given. After the first meeting students had some days to decide on the focus of their solution and presentation, which was then discussed in class. Students decided on how to organize their work and how to split the workload within the teams. Students also had the chance to ask for more assistance and clarifications that could help them working through their proposed solution.



In the case of Poland

Preparation actions: Preparations began one month in advance. Students had to get acquainted with the task presented by the tourist office. It consisted in constructing an algorithm allowing the creation of a trip plan for students visiting Krakow. This plan had to take into account a number of conditions, such as: budget, duration, age of participants, choice of optional attractions and others.

Students preparing their MID-DAY presentations: As the event was attended by groups of students from the same classes, the preparations took place in the stationary version. Their work consisted in collecting relevant data, such as: ticket prices, distance between points, coach rental rates, public transport prices and others. In the second phase of preparations, their task was to create a program to simulate the costs of the event. Both traditional methods and modern information technologies were used for the preparation. Some of the work was also done remotely. In the final, pre-event phase, group members concentrated on presenting their achievements, namely preparing and training their speeches.

The actual MID-DAY: The event was divided into three parts: In the first part, the INNOMATH project, its goals, assumptions and effects developed during its duration, were presented. The second part consisted of the presentation of students' works. All groups presented the results of their preparations. After each of them, experts and invited guests had the opportunity to express their opinions on the prepared shows. The third part was a discussion on topics related to the project and its impact on the modern approach to teaching.



In the case of Romania:

Note: This example of MID-Day is written from the perspective of the students

Throughout the course of the INNOMATH project we got to experience many new things that showed us what life is like beyond high school and how math can be used in real world situations. In order to better understand how quintessential applied mathematics is for the smooth operation of businesses and industrial facilities, we set off on a journey to try to solve an issue a local company was facing, by utilizing our knowledge.

Thus, we visited Fluorocarbon Polymers and spoke with its director. He explained to us that they produce and ship various pieces and materials which serve all sorts of purposes in industry. We then proceeded to follow the production line from start to finish, hoping to spot ways in which the company loses money and/or resources so we could come up with a solution. It proved quite challenging to find issues in their process, but in the end, we decided to try to optimize the transport of the raw materials they use and of the finished products to their buyers.

Transport optimization problems are difficult to solve and require university-level skills, therefore what we ended up doing is learn some advanced algebra that only brought us closer to being able to solve such problems, while also helping us solve high school-level linear algebra problems more easily.

In the classroom our teacher divided us into two groups: one which would learn and use the methods usually taught in high school for computing inverse matrices, determining the rank of a matrix, and solving systems of linear equations; and one which would learn the advanced methods. The aim was to compare the different methods at the end of the course and determine which are faster and easier to use.

As we practiced solving exercises, we were encouraged to help each other understand and learn more quickly. Indeed, sometimes it felt easier to ask my classmates questions about the new concepts rather than ask the teacher, and their perspective as pupil allowed them to have a certain useful insight into the problem. The most enjoyable aspect of this learning method was the fact that it felt less like learning and more like playing as we were bouncing ideas off one another.

In the end, both teams presented and swapped the methods they had used to get a feel for the other half of this story and be able to draw conclusions more easily. We all agreed that the advanced methods were not as hard to understand as we had thought previously, but it really matters how you learn them. Looking back at all we have seen and achieved, I feel good about choosing to study engineering in university and better prepared to face the real world.

The actual MID-DAY:

The image shows a Zoom meeting interface. The main window displays a presentation slide titled "Ce este MID-Day?". The slide includes the "meomath" logo, the text "Co-funded by the Erasmus+ Programme of the European Union" with the EU flag, and a photograph of students in a classroom. The slide text reads: "Ce este MID-DAY? MID-Day: o zi reală a matematicii și a industriei, când elevii abordează o problemă stabilită de un partener industrial și analizată / adaptată de academicieni / cercetători și facilitată de profesori." The project number "2019-1-DE03-KA201-059604" is visible at the bottom of the slide.

Below the slide, a gallery view shows several participants in a grid. The names visible are Nicușor Voinea, Cristian Nasta, Barni, Preda Mircea, NICOLETA MADRINSCHI, and MATTEO SALIANI. A "You" tile is also present. The meeting title at the bottom of the Zoom window is "1:02 PM | MID-DAY: Matematica întâlnește Industria".

1.8: Evaluation of the MID-Day events: Comments by students

After completing the MID-Day events in the five Countries mentioned above the participants of the event were sent an evaluation to assess their satisfaction on the MID-Day activities.

The overall feedback from students was very positive and encouraging. Students expressed their satisfaction for having the opportunity to work as a team member and being assigned roles within their teams. They believed that it was enjoyable and constructive. All students from all the events said that they benefitted from the MID-Day activity for various reasons. One of the reasons was that they worked in a real-life problem as a member of a team that they developed their communication and presentation skills, that they realized how complicated real-life problems are and how mathematics can provide a solution for them. Students also mentioned that the opportunity to present their work during the MID Day format turned out to be important for them - thanks to which they could improve and develop their social skills such as: information exchange, argumentation. Finally, all students mentioned that it was a unique experience for them to meet the industry world!

What is important to mention is that all students from all countries agreed that the MID-day problems were a much more interesting and interactive way to learn than the mathematical problems they face in their schools. In addition, the majority of students mentioned that they managed to learn something new from the MID-Day event.

When students were asked about their opinion on the advantages, practicality and difficulties of adopting this in the school curricula, some said that it is necessary to have this kind of problems in the school curricula and that students will gain more confidence about Mathematics working in teams and gain real working experience. They believe that it would not be difficult to adopt this culture in the school. On the other hand, some students expressed the opinion that although for them this was a great experience, that mathematics school level is low and that very nice experience that in schools the interest in mathematics is low and bigger tasks like this could further alienate some students from Mathematics.

As for some ideas towards the improvement of the MID-Day students would like to see more problem options in the future with varying difficulty. They found the problems and the whole process very challenging, and they expressed their wish for such types of events and activities to be included in the school curricula where possible.

Teil C: Das Unterrichtsmodell "Mathematik trifft Industrie in der Schule" - Leitlinien für die Durchführung eines „MID-Days“ (Tag der Mathematik und Industrie) (DE)

Kapitel 1: Leitlinien für einen MID-Day

Zunächst muss man die begabten Schüler identifizieren und dann im nächsten Schritt die Industrie davon überzeugen, dass deren Fähigkeiten zum unkonventionellen Denken in betriebswirtschaftliche und industrielle Prozesse involviert werden sollten. Gleichzeitig müsse man Akademiker und Lehrer mobilisieren, die sie unterstützen, einen MID-Tag zu planen und gestalten. In diesem Kapitel werden die Leitlinien für die Planung und Durchführung einer solchen Veranstaltung vorgestellt.

1.1: Definition und Glossar

Partner: Wenn wir in diesen Leitlinien von „Partnern“ sprechen, meinen wir einen Betrieb, der dazu bereit ist, ein oder zwei Tage in Gespräche mit Akademikern, Lehrern und Studenten zu investieren, um ein wirtschaftliches Problem zu definieren und zu lösen.

Wirtschaftliche Fallstudien: Ein geeignetes Thema, das vom Industriepartner vorgebracht wird und für Studierende, Lehrkräfte und Akademiker verständlich ist. Es muss von beiden Seiten angepasst werden, einige nützliche Fragen auf Seiten der Industrie ansprechen und für die Studierenden eine Herausforderung darstellen, die jedoch machbar ist. Die Problemstellung ist das Ergebnis eines gemeinsamen Brainstormings zwischen dem Industriepartner, den Akademikern und den Lehrern im Vorfeld des MID-Day. Eine Liste möglicher Probleme findet sich in Abschnitt 1.4 unten.

MID-Day: Ein tatsächlicher Mathematik- und Industrietag, an dem die Schüler ein von einem Industriepartner gestelltes und von den Akademikern und Lehrern angepasstes Problem lösen. Dabei kann es sich eher um eine Ideenfindung, eine Präzisierung einer Idee, als um die vollständige Lösung eines Problems handeln.

1.2: Methodik, Anforderungen und Leitlinien für Lehrkräfte (der Lehrer als Mentor und Mitgestalter)

Die Verantwortung der Lehrer besteht darin, die Schüler so auszuwählen und vorzubereiten, dass sie motiviert an einem fruchtbaren MID-Day teilnehmen und gestellte Aufgaben ohne zu hohe Überforderung lösen können. Am MID-Day sollten die Lehrer die Arbeit der Schüler erleichtern. Das bedeutet, dass Sie zunächst die Schüler auswählen müssen, sei es mit Hilfe eines schriftlichen Tests oder auf der Grundlage einer Vorauswahl durch den Elternbeirat, der Schulnoten, der Auswahl für die Olympiade oder die Mathe-Clubs. Außerdem müssen Sie von den Eltern und den Schulen formale Vereinbarungen für die Teilnahme aller einholen.

Nach einem ersten Gespräch mit dem Industriepartner und den Akademikern müssen Sie sich für die Art des MID-Tages entscheiden, den Sie durchführen möchten. Nachstehend finden Sie einige Vorschläge. Sie sollten auf der Grundlage eines von uns zur Verfügung gestellten Lehrplans einen Grundkurs einrichten, um die Schüler auf ein angemessenes Niveau zu bringen. Dies bedeutet auch, dass die Rollen je nach Niveau angepasst werden müssen: Einige fortgeschrittenere oder reifere Schüler können als Coaches oder Tutoren für die Jüngeren fungieren, als *guide on the side* anstatt als *sage on the stage*.

Bei der Moderation des MID-Day geht es darum, eingeschüchterte Schüler zu unterstützen, ihr Selbstvertrauen zu stärken, ihnen zu versichern, dass ihr Wissen nicht unzureichend ist, um über das Problem zu reflektieren, und die Kommunikation zu erleichtern, dafür zu sorgen, dass sich niemand unsicher fühlt oder Meinungen aufgedrängt werden, dass jede Stimme gehört wird, dass sich jeder Schüler auf die eine oder andere Weise gebraucht und geschätzt fühlt. Die Aufteilung des Problems in Teilaufgaben muss zu diesem Zweck im Voraus geplant werden. Als letzten Schritt sollten Sie Feedback darüber geben, wie das Projekt bewältigt wurde, indem Sie die (Eigen-)Initiative und Reaktionen der Schüler beobachten sowie motivationsfördernde Aspekte, deren Mitarbeit, deren Vorschläge und geschaffene Produkte entsprechend honorieren.

Nach dem MID-Day sollten Sie besonders darauf achten, wie die während diesem erarbeiteten und angesprochenen Inhalte im alltäglichen Mathematikunterricht weiter verwendet werden können, indem Sie es mit dem Schulwissen aus dem Unterricht in Verbindung bringen. Suchen Sie außerdem nach Gelegenheiten, während welcher die Schüler ihre Arbeiten auf Messen, in Wettbewerben, Foren, Clubs oder bei der Einrichtung einer Ausstellung in Ihrer Schule mit anderen teilen können. Dies wird dazu beitragen, Schüler für das kommende Jahr zu begeistern. Vergessen Sie nicht zu erwähnen, dass die Teilnahme der Schüler in ihren Lebenslauf einfließt, wenn sie sich um Stipendien bewerben. Diese Veranstaltung wird ihr berufliches Netzwerk und ihre Selbstsicherheit beleben und fördern!

1.3: Methodik, Anforderungen und Leitlinien für Hochschulen, die MID-Tage unterstützen

Die Partneruniversitäten und -hochschulen sollten ein Team von Studenten zusammenstellen, das an dem Industriefall arbeitet. Das bedeutet, dass sie mit den Studenten zusammenarbeiten und diese ebenso im Bereich Teamarbeit schulen. In Zusammenarbeit mit den Industriepartnern und Lehrkräften wird das Problem so gestaltet, dass es von den Studierenden bzw. Schülern bearbeitet werden kann.

Ihre Hauptaufgabe besteht darin, den MID-Day und die Vorbereitungssitzungen davor zu gestalten, zu organisieren und zu leiten. Eine typische Interaktion mit Lehrern und Schülern besteht aus drei Phasen:

Das erste Treffen: Sie lernen die Schüler und ihre Lehrer kennen, motivieren sie und bilden ein Team unterstützt von originellem, kreativem und unterhaltsamem mathematischen Materials. Sie können Teilnehmer für den Mathe-Club, die Wissenschaftsmesse, den Mathe-Tag, Mathewettbewerbe oder schulübergreifende Treffen mobilisieren.

Das zweite Treffen: Ausgewählte Schüler werden dazu animiert, Teams zu bilden. Deshalb müssen Ihnen Werkzeuge zur Ausbildung und Förderung des Teamworks und der Kommunikation im Team an die Hand gelegt werden.

Das dritte Treffen besteht aus dem *eigentlichen* MID-Day. Sie müssen geeignete und ausreichend große Räumlichkeiten für das Treffen auswählen, sei es an der Universität oder am Standort des Industriepartners, für Kaffeepausen und Mittagessen sorgen oder Ähnliches. Eine Hybridversion mit Onlinekonferenzen ist bei Unmöglichkeit der Durchführung eines physischen Treffens angebracht.

Sie müssen Fähig- und Fertigkeiten, die Teamwork und Kommunikation in der Gruppe erleichtern, fördern, und gezielt Fragen stellen. Achten Sie darauf, dass Sie nicht selbst nach Antworten suchen sondern den Schülern diese Möglichkeit einräumen. Stellen Sie sicher, dass sich die Schüler wichtig und ganzheitlich wahrgenommen fühlen und im Zentrum des MID-Days stehen. Sie sind nicht der Funktion des „belehrenden Lehrers“ sondern als auszubildender Forscherkollege.

Grundlegende Werkzeuge für die Arbeit im Team:

- Divergenzen deutlich machen. Unter Wahrung eines sicheren Umfelds eine fruchtbare Debatte führen: nicht abstimmen, keine Kompromisse eingehen, nicht willkürlich auswählen, Mittelwertbildung und Feilschen vermeiden, sondern Einstimmigkeit anstreben.
- Zuhören und einbeziehen: Lassen Sie niemanden seinen Standpunkt aufzwingen, sondern argumentieren Sie, weisen Sie nicht ab, sondern schlagen Sie Gegenargumente vor.
- Nehmen Sie Debatten nicht persönlich, nehmen Sie Dinge nicht persönlich. Suchen Sie nach Lösungen.
- Akzeptieren Sie Lösungen nur, wenn sie von allen verstanden und geteilt werden.
- Ein leichtes Training zu diesen Themen, sowohl für Schüler als auch für Lehrer, erhöht die Wahrscheinlichkeit, dass originelle Lösungen und ein produktives Team entstehen.

1.4: Vorgeschlagene Beispiele für MID-Tage

- **Diskutieren Sie mit Ihrem Industriepartner.** Optimierungsprobleme sind in vielen Unternehmen zu finden: Ganz gleich, ob der Industriepartner Glas schneidet, Roboter numerisch steuert, ein Lagerhaus verwaltet oder eine Lieferkette organisiert, er hat sicherlich ein grundlegendes offenes Optimierungsproblem, das geklärt, wenn nicht gar gelöst werden muss.
- **Eine statistische Untersuchung** wird Ihren Industriepartner interessieren, denn er möchte vielleicht die Bedürfnisse seiner Kunden oder Mitarbeiter besser kennenlernen, und ein Markt- oder Personalforschungsprojekt ist relativ einfach einzurichten, macht Spaß, ist kreativ zu gestalten und kann zu unerwarteten Ergebnissen führen, wenn man es von einem ungewöhnlichen Standpunkt aus betrachtet: "Wie kann man diesen trostlosen offenen Raum in einen gemütlichen Kontext betten?". Bevor es ans Eingemachte geht, ist eine Lektion in Statistik und Erhebungsmethodik von Nöten.
- **Ein wissenschaftlicher Lehrpfad bzw. Stationenarbeiten** auf dem Industriegelände sind gute Möglichkeit für die Industrie, ihre Arbeit während der Unterrichtsbesuche vorzustellen, und für die Schüler, zu Problemlösern zu werden und ihre Umgebung mit einem wissenschaftlichen Auge zu betrachten: "Wie groß ist dieser Tank? Wie lange dauert es, ihn zu füllen?" Der Unterrichtsplan sollte sich auf das Modellieren und die Messmethodik konzentrieren, indem man zum Beispiel entdeckt, dass der eigene Körper ein ziemlich präzises Werkzeug für die Schätzung von Alltagsgrößen ist.
- Wenn Sie keinen direkten Kontakt zu einem Industriepartner haben, **denken Sie auch an die Bildungsindustrie** und lassen Sie die SchülerInnen pädagogische Spiele für Sie entwickeln: Lassen Sie sie neue Wege für den Unterricht erfinden, indem sie einige Ihrer Inhalte spielerisch gestalten. Lernen Sie von ihnen! Der Unterrichtsplan sollte die neuronalen Grundlagen der Kognition, die Gamifizierung, die Festlegung von Zielen und Unterzielen, die Definition von Herausforderungen und Niveaus, die Möglichkeiten der Zusammenarbeit und das bessere Verständnis des eigenen kognitiven Prozesses durch die SchülerInnen vermitteln. Und man versteht ein Konzept nie besser, als wenn Schüler in die lehrende Rolle schlüpfen.

Sobald das Hauptthema festgelegt ist, können Sie den Unterrichtsverlauf so planen, dass die Schüler von ihrem Ausgangspunkt aus durch die Bearbeitung der Aufgaben zu den erforderlichen Kompetenzen gelangen. Das heißt: Sie müssen in den Unterricht eine Einführungssitzung in das Projekt integrieren. Diese Erstkonfrontation sollte in einer positiven und lernfördernden Atmosphäre stattfinden. Die Schüler arbeiten in Teams und lernen sich gegenseitig kennen.

Sobald der Unterrichtsfall mit dem Partner ausgearbeitet ist, sollte er den Schülern im Voraus zugesandt werden um ihnen die Möglichkeit zu geben, sich inhaltlich mit dem Thema auseinanderzusetzen und Fragen zu stellen.

Am Tag des MID-Day besuchen die Schüler den Industriestandort, diskutieren zumindest mit Vertretern der Industrie und arbeiten am Vormittag gemeinsam an ihrem Projekt.

Hier ist ein möglicher Zeitplan für den MID-Day:

- 9h-9:30: Begrüßung, Smalltalk, Eisbrecher
- 9:30-10:30: Teambildung mit einer Gruppenaktivität wie "Lost on the Moon" oder ähnlich.
- 10:30-10:45: Pause
- 10:45-11:45: Diskussion in Gruppen über das Problem des MID-Day, Sicherung des Verständnisses über das Themas, erste Gedankengänge
- 11:45-12:15: Erste Synthese möglicher Strategien zur Bewältigung der Probleme, Festlegung von Gruppen für spezifische Aufgaben.
- 12:15-13:45: Mittagspause
- 13:45-15:45: Arbeit in Untergruppen an spezifischen Strategien und Aufgaben
- 15:45-16: Pause
- 16-16:30: Vorbereitung der Synthese und ihrer Präsentation [Die Studenten]
- 16:30-17:15: Präsentation vor den Industriepartnern, Diskussion

1.5: Evaluationsergebnisse, Do's und Don'ts

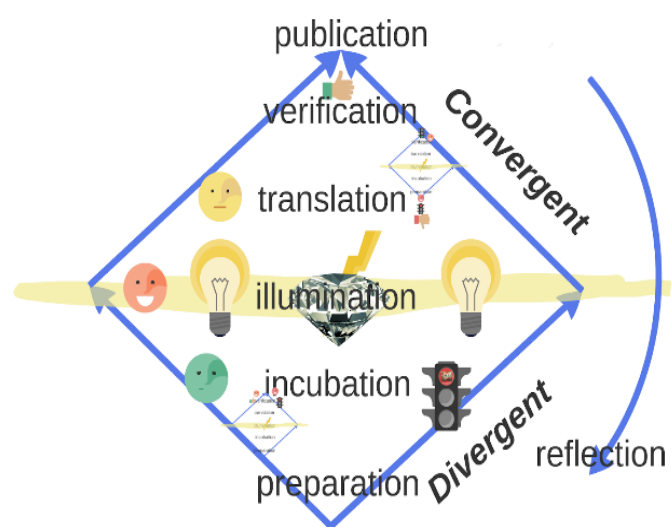
Während des MID-Day:

Achten Sie darauf, dass jeder Schüler einer kleinen Gruppe von höchstens 8 Mitgliedern angehört, in denen vorbereitenden Aktivitäten durchgeführt werden, z. B. mit älteren Schülern, die die Gruppe als Coach betreuen, so dass sich jeder gebraucht und zugehörig fühlt. Man muss unsicheren Schülern besondere Aufmerksamkeit schenken, indem man geeignete Fragen stellt, die besonderen Fähigkeiten jedes Schülers hervorhebt und entwickelt und das Selbstvertrauen stärkt. Das industrielle Problem wird so unter Berücksichtigung aller Perspektiven analysiert, so dass es möglich ist, Teilaufgaben unter allen Teammitgliedern aufzuteilen und die Zusammenarbeit aller zu fördern.

Die Bewertung sollte sich auf die Aspekte konzentrieren, die den Schüler in seiner Gesamtheit als individueller Denker und Schöpfer erfassen. Natürlich können nicht alle Ideen zu einem erfolgreichen Produkt führen, womöglich wird auch keine Produktion aus dem MID-Day hervorgehen. Dies sollte nicht als Misserfolg empfunden werden, die Potenziale und die Kreativität der Ideen und Ergebnisse sollten dennoch hervorgehoben werden. Wenn Sie Zahlen benötigen oder individuelle Preise vergeben wollen, können Sie die Bewertung der Kreativität des stattgefundenen Ideenfindungsprozesses entlang der von Guilford beschriebenen

Dimensionen objektivieren: *fluidity* („Flüssigkeit“), *flexibility* („Flexibilität“), *elaboration* („Ausarbeitung“) und *originality* („Originalität“):

- *fluidity* meint die Fähigkeit, viel zu produzieren und viele Beispiele auszuarbeiten.
- *flexibility* ist die Fähigkeit, eine Strategie zu ändern, wenn man nicht weiterkommt; Glück und das Serendipitätsprinzip sind hier eindeutig die Schlüsselwörter.
- *elaboration* geht einer Frage auf den Grund, sucht nach den feinsten Details und ist erschöpfend in ihrer Suche.
- *originality* ist die Art von Kreativität, die man erkennt, wenn man sie sieht: So etwas hat man noch nie gesehen, es ist neu und provokativ



Um wirklich kreativ zu sein, muss eine Idee den gesamten Kreativitätszyklus mit den beiden Phasen, der Divergenz und der Konvergenz, durchlaufen: von der Vorbereitung über die Inkubation bis hin zur Erleuchtung (dem Aha-Erlebnis). Jedoch muss diese Idee über den Kopf und die kognitive Ebene des Schülers hinweg in Worte, Skizzen usw. umgesetzt werden, um dann eine Überprüfung und einen Austausch unter den Mitschülern und Studenten bzw. den Projektpartnern zu ermöglichen. Am Ende kann diese Idee reflektiert und in einen größeren Ideenkreislauf integriert werden, um das angesprochene Problem zu lösen.

Man sollte Kreativität nicht mit divergentem Denken verwechseln: Eine Brainstormingsphase ist in Ordnung, aber irgendwann muss sie sich von Tagträumereien lösen und zu einer tatsächlich praktikablen Lösung konvergieren, und in dieser konvergenten Phase ist ebenso viel Kreativität gefragt. Ebenso darf der Aha-Moment der Einsicht nicht überbewertet werden, Poesie und Industrie können sich nur bis zu einem gewissen Punkt vermischen, und was wie eine originelle Idee aussieht, muss sich in konkrete Pläne umsetzen lassen. Diese subtilen Punkte müssen vom Team berücksichtigt werden, um produktive kognitive Prozesse unter den Schülern zu fördern.

Nach dem MID-Day:

Nicht alle Ergebnisse werden sofort während des MID-Days sichtbar. Die Schüler werden mit vielen Beobachtungen, Ideen und Plänen nach Hause kommen. In der letzten Phase können diese Arbeiten überarbeitet und dem Industriepartner erneut präsentiert werden. Es ist wichtig, eine Möglichkeit zu schaffen, dass sie sich mit anderen austauschen können. Dies kann für Schulmessen, Foren, Schulpartnerschaften, Integrationswochen, Wettbewerbe und so weiter genutzt werden, um zu zeigen, was man erreicht hat.

1.6: Warum sich die Industrie engagieren sollte, Vorteile für die Industrie

Dieser Aspekt seitens des Industriepartners muss im Vorfeld geklärt werden, dennoch ist es wichtig, ihn am Ende des MID-Day noch einmal aufzugreifen und folgende Checkliste durchzugehen.

Haben alle Beteiligten von dem MID-Day profitiert? Hat die konventionelle Denkweise wirklich ein industrielles Problem gelöst, oder hat die Industrie einfach nur ihr Geschäft nach außen hin darstellen können. Der Nutzen ist je nach Art des durchgeführten MID-Day sehr unterschiedlich. Um die allgemeinen Beispiele durchzugehen, haben wir sie genannt:

- Konnte die statistische Erhebung Elemente sichtbar machen, die vorher nicht zu sehen waren, oder war sie einfach ein Mittel zur Kommunikation, sei es intern oder mit potenziellen Kunden? Den Kunden oder besser den Mitarbeitern die Möglichkeit zu geben, ihre Meinung zu äußern und zu sehen, dass sie berücksichtigt wird, ist bereits schätzenswert.
- Macht der Lernpfad Spaß und ist informativ? Kann er von potenziellen Kunden oder von Schulklassen genutzt werden? Öffnet er den Teilnehmern die Augen für die Herausforderungen, mit denen sich die Branche auseinandersetzt? Entdeckt der Partner im Falle einer Exkursion zu einer Stätte diese mit anderen Augen? Erfüllt sie ihren wissenschaftlichen Zweck?
- Haben die Schüler interessante, informative und unterhaltsame pädagogische Spiele für die Bildungsbranche entwickelt oder durchgeführt? Haben sie bei der Gestaltung des pädagogischen Spiels etwas gelernt?
- Die Industrie wird die Möglichkeit haben, mit begabten Schülern und Studenten in Kontakt zu treten und möglicherweise in Zukunft mit ihnen zu arbeiten.

1.7: Durchführung eines MID-Day-Events in verschiedenen Ländern:

Der MID-Day wurde in fünf europäischen Ländern erprobt und durchgeführt: Zypern, Frankreich, Deutschland, Polen und Rumänien. Die Idee hinter der MID-Day-Veranstaltung ist, dass die praktische Anwendung von Mathematik im realen Leben heutzutage ein absolutes Muss für jeden Lehrplan ist, und noch mehr für begabte Schüler, die zu Innovationen beitragen können und die das Leben der Menschen verbessern könnten sowie gleichzeitig den Unternehmergeist in den Lehrplänen fördern. Die Veranstaltung "Mathematik trifft Industrie" soll den Schülerinnen und Schülern herausfordernde, ungelöste reale Probleme aus der Industrie bieten, mit dem Ziel, entweder in Teams oder einzeln zu arbeiten und die Mathematik zu nutzen, um eine Reihe von möglichen Lösungen für die gegebenen Probleme vorzuschlagen. Darüber hinaus hatten die SchülerInnen die Möglichkeit, miteinander zu kooperieren und auf ein gemeinsames Ziel hinzuarbeiten, HochschulforscherInnen sowie Menschen aus der Industrie zu treffen. LehrerInnen der Sekundarstufe hatten die Möglichkeit, Ideen zu sammeln und sich inspirieren zu lassen, wie solche Probleme als Teil ihres regulären Unterrichts umgesetzt und verwendet werden könnten.

Die Vorbereitung und Durchführung der Veranstaltung variierte von Land zu Land. Es wurden verschiedene Probleme aus der Industrie vorgestellt, die die SchülerInnen lösen sollten. Daher werden wir im Folgenden die Vorbereitung und Durchführung jeder MID-Day-Veranstaltung nach Ländern analysieren. Auf diese Weise können Beispiele und Anregungen für die MID-Day-Methodik sowie Didaktik gewonnen werden.

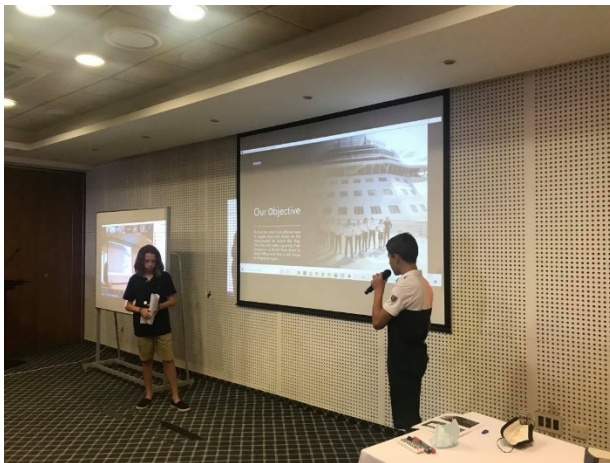
Cyperns MID-Day

Vorbereitungsmaßnahmen: Die Vorbereitungen für diese Veranstaltung begannen zwei Monate im Voraus, als ein Einladungs-/Motivationsschreiben an SchülerInnen verschickt wurde, die Spaß an Mathematik haben und an einer solchen Aktivität interessiert sein könnten. In der Zwischenzeit nahm das INNOMATH-Lehrerteam der Heritage Private School nach der Organisation einer Fokusgruppe Kontakt zu lokalen Unternehmen auf und bat sie, sich an dieser Veranstaltung zu beteiligen, indem sie den Schülern ein reales Problem aus der Industrie zur Verfügung stellten. Zwei Unternehmen stellten jeweils ein Problem für den MID-Day zur Verfügung. Problem A wurde von *V-ships*, einem der größten Schifffahrtsunternehmen der Welt, bereitgestellt. Das Problem beschrieb den konkreten Fall eines Tankschiffs, das zwischen Singapur und Brasilien verkehrt und mit Vorräten versorgt werden muss, um jederzeit welche an Bord zu haben. In diesem Projekt mussten die Studenten im Interesse des Catering-Unternehmens handeln und versuchen, den kosteneffizientesten Plan für den Proviantnachschub zu formulieren, indem sie sicherstellen, dass die vereinbarten Preise eingehalten werden, das Schiff während der Reise jederzeit genügend Vorräte an Bord hat und sie gleichzeitig den Gewinn für das Catering-Unternehmen maximieren mussten. Aufgabe B wurde von einem Architekturbüro gestellt und die Schüler wurden gebeten, den Platz eines Architekten einzunehmen, den Prozess zu verstehen und die bestmögliche Lösung für einen Kunden zu finden, die auf dessen Hausanforderungen, den zu verwendenden Materialien und dem verfügbaren Budget basiert.

Schüler bereiten ihre MID-DAY-Präsentationen vor: Viele mathematikbegeisterte Schüler haben sich für die Teilnahme am MID-Day entschieden. Es war eine Gelegenheit für sie, kreativ zu denken und über den Tellerrand hinauszuschauen, entweder einzeln oder in Teams, um mögliche Lösungen für die vorgegebenen Probleme in Aktivitäten außerhalb des Klassenzimmers zu finden. Aufgrund der Einschränkungen des Covid19 und der Tatsache, dass die teilnehmenden SchülerInnen aus verschiedenen Städten kamen, wurden zwei Online-Treffen für die LehrerInnen der Heritage-Schule, die teilnehmenden SchülerInnen, die UnternehmensvertreterInnen und die Mathematical Society organisiert. Die Online-Treffen fanden Anfang und Mitte Juli statt. Ziel der Treffen war es, dass die Schülerinnen und Schüler ihre Teams zusammenstellen, sich mit den Aufgaben vertraut machen und zusätzliche Erklärungen und Erläuterungen von den Lehrkräften und den Unternehmensvertretern erhielten. Außerdem wurden allgemeine Leitlinien für den reibungslosen Ablauf des MID-Day und die Anforderungen an die Präsentationen der Schüler gegeben. Nach dem ersten Treffen hatten die Schüler einige Tage Zeit, um sich für ihr bevorzugtes Problem zu entscheiden, und begannen mit der Arbeit daran. Fallstudie A war beliebter, denn sie wurde von allen Studierenden ausgewählt. Dann fand eine weitere Online-Sitzung für SchülerInnen und LehrerInnen statt, um den Fortschritt bei der Entwicklung der Lösung zu besprechen. Die SchülerInnen wurden beraten, wie sie ihre Arbeit organisieren und

wie sie das Arbeitspensum innerhalb des Teams aufteilen sollten. Die SchülerInnen hatten auch die Möglichkeit, weitere Unterstützung oder Erklärungen zu erhalten, die ihnen bei der Ausarbeitung ihrer vorgeschlagenen Lösung helfen könnten.

Der eigentliche MID-DAY: Am Mittwoch, dem 28. Juli 2021, fand die Veranstaltung in Agros in Anwesenheit von Schülern, Eltern, Lehrern, Hochschulforschern und Projektpartnern statt. Die SchülerInnen präsentierten ihre möglichen Lösungen und am Ende jeder Präsentation gab der Vertreter von V-ships, Herr Loucas Chrysochos, sein Feedback zu den von den SchülerInnen vorgeschlagenen Lösungen ab. Auch alle anderen TeilnehmerInnen konnten Fragen an die SchülerInnen stellen. Alle von den Schülern vorgeschlagenen Lösungen waren unterschiedlich, was zu einer fruchtbaren Diskussion zwischen den teilnehmenden Studierenden und dem Publikum führte. Am Ende äußerte sich der Vertreter aus der Industrie begeistert über die harte Arbeit der SchülerInnen, ihre ausgezeichneten Präsentationsfähigkeiten und die analytische Art und Weise der Lösungsvorstellung. Er erwähnte, dass es für ihn eine großartige Erfahrung war, an dieser Veranstaltung teilzunehmen und solche Schüler zu treffen, die eine großartige Arbeit geleistet haben. Außerdem bekundete er sein Interesse daran, eines Tages mit ihnen zusammenzuarbeiten.



Frankreichs MID-DAY

Der eigentliche MID-DAY:

Die Schüler, die am MID-Day teilnahmen, trugen zwei Hüte: Sie waren sich bewusst, nicht nur teilzunehmen, sondern auch das innovative pädagogische Konzept zu testen. Die Schüler wurden in eine kreative Denkweise versetzt, in der Mathematik als konkretes Werkzeug zur Lösung realer Probleme mit wirtschaftlichem Wert betrachtet wurde.

Sie durften sich mit Hilfe von Eisbrechern in Teams zusammenfinden, dann die MathCityMap-Anwendung installieren und den lokalen Matheparcours ablaufen.

Den größten Teil des Tages verbrachten sie im UNESCO-geschützten Renaissance-Viertel, lauschten den Geschichten des professionellen Führers, machten sich Notizen, arbeiteten im Team, probierten Ideen aus und ließen andere Teammitglieder die Machbarkeit und den Spaß an den Aufgaben überprüfen. Zurück an der Universität wurden die Aufgaben im MathCityMap-System kodiert.

Das Hauptproblem bestand darin, den Bedürfnissen des Vertreters der Tourismusbranche gerecht zu werden: Die Studierenden diskutierten, wie sie am besten verschiedene Zielgruppen ansprechen können wie Familien, Klassenverbände und Freundesgruppen. Das Hauptziel des erstellten mathematischen Lehrpfades ist es, die Perspektive des professionellen Reiseleiters zu verstehen und genügend unterhaltsame und augenöffnende Informationen in wissenschaftliche Fragen einzubauen.

Der zweite Teil des MID-Day war das Feedback an den Industriepartner. Die Schülerinnen und Schüler hatten die Möglichkeit, in Teams ihren Vortrag vorzubereiten. Dies geschah während des transnationalen Projekttreffens, das Mitte Dezember 2021 in Lyon auf dem UCBL-Campus stattfand. Am Tag danach begründeten die Schülerinnen und Schüler ihre Entscheidungen vor der Jury, die sich aus einem Vertreter der Industrie und den internationalen Partnern des InnoMath-Projekts zusammensetzte.



Deutschlands MID-Day

Vorbereitung: Die Vorbereitungen für die MID-DAY-Veranstaltung begannen sechs Monate im Voraus mit einem Treffen von Industriepartnern, während welchem mögliche Aufgaben besprochen wurden. Die Fokusgruppe, die sich aus Lehrern, Wissenschaftlern und Industriepartnern zusammensetzte, traf sich und bewertete die Ideen und den vorgeschlagenen Zeitplan für den MID-DAY. Später wurden auch Schulen mit mathematisch-naturwissenschaftlichem Schwerpunkt eingeladen, sich an der Veranstaltung zu beteiligen. Gemeinsam mit der Stiftung Planetarium Berlin wurde beschlossen, den Schülerinnen und Schülern die Frage zu stellen, wie die Veränderung der Jahreszeiten - der Klimawandel - statistisch sichtbar gemacht werden kann und wie Vorhersagen getroffen werden können. Dabei sollten insbesondere die verwendeten Modelle bewertet und reflektiert werden.

Schüler bereiten ihre MID-DAY-Präsentationen vor:

14 Klassen (420 Schüler) von Schulen mit mathematisch-naturwissenschaftlichem Schwerpunkt nahmen an der MID-DAY-Aktivität teil. Aufgrund der Beschränkungen durch Covid-19 mussten sich die Schüler in ihren Schulen treffen und konnten bei der Vorbereitung der Präsentation nicht mit Schülern anderer Schulen zusammenkommen. Die Vorbereitungen fanden im September statt, und die Schüler erhielten Material, um sich mit den Problemen vertraut zu machen, das von den Forschern und den Industriepartnern vorbereitet worden war. Die Lehrer unterstützten die Schüler und beaufsichtigten sie. Es wurden auch allgemeine Richtlinien für den reibungslosen Ablauf des MID-DAY und die Anforderungen an die Schülerpräsentationen gegeben. Nach dem ersten Treffen hatten die Schüler einige Tage Zeit, um den Schwerpunkt ihrer Lösung und ihrer Präsentation festzulegen, der dann in der Klasse besprochen wurde. Die Schüler entschieden, wie sie ihre Arbeit organisieren und wie sie die Arbeitslast innerhalb der Teams aufteilen wollten. Die SchülerInnen hatten auch die Möglichkeit, um weitere Unterstützung und Klarstellungen zu bitten, die ihnen bei der Ausarbeitung ihrer vorgeschlagenen Lösung helfen könnten.



Polens MID-Day

Vorbereitungsmaßnahmen: Die Vorbereitungen begannen einen Monat im Voraus. Die Schüler mussten sich mit der vom Fremdenverkehrsamt gestellten Aufgabe vertraut machen. Sie bestand darin, einen Algorithmus zu erstellen, der es ermöglichte, einen Reiseplan für Schüler zu erstellen, die Krakau besuchen. Dieser Plan musste eine Reihe von Bedingungen berücksichtigen, wie z.B. Budget, Dauer, Alter der Teilnehmer, Auswahl der optionalen Attraktionen und andere.

Schüler bereiten ihre MID-DAY-Präsentationen vor: Da die Veranstaltung von Schülergruppen aus denselben Klassen besucht wurde, fanden die Vorbereitungen in einer stationären Variante statt. Ihre Arbeit bestand darin, relevante Daten zu sammeln, wie z. B. Fahrkartenpreise, Entfernungen zwischen den einzelnen Orten, Mietpreise für Busse, Preise für öffentliche Verkehrsmittel und andere. In der zweiten Phase der Vorbereitungen bestand ihre Aufgabe darin, ein Programm zu erstellen, mit dem die Kosten der Veranstaltung simuliert werden konnten. Bei der Vorbereitung wurden sowohl traditionelle Methoden als auch moderne Technik und IT eingesetzt. Ein Teil der Arbeit wurde auch aus der Ferne erledigt. In der letzten Phase vor der Veranstaltung konzentrierten sich die Gruppenmitglieder auf die Präsentation ihrer Leistungen, d. h. auf die Vorbereitung und das Einüben ihrer Referate.

Der eigentliche MID-Day:

Die Veranstaltung war in drei Teile gegliedert: Im ersten Teil wurden das INNOMATH-Projekt, seine Ziele, Annahmen und Auswirkungen, die sich während seiner Laufzeit entwickelt haben, vorgestellt. Der zweite Teil bestand aus der Präsentation der Schülerarbeiten. Alle Gruppen präsentierten die Ergebnisse ihrer Vorbereitungen. Anschließend hatten Experten und geladene Gäste die Möglichkeit, ihre Meinung zu den vorbereiteten Präsentationen zu äußern. Der dritte Teil bestand aus einer Diskussion über Themen im Zusammenhang mit dem Projekt und seinen Auswirkungen auf den modernen Unterrichtsansatz.



Rumäniens MID-Day:

Anmerkung: Es handelt sich hierbei um die Schilderung aus Schülerperspektive

Im Laufe des INNOMATH-Projekts haben wir viele neue Erfahrungen gemacht, die uns gezeigt haben, wie das Leben jenseits der Schule aussieht und wie Mathematik in realen Situationen eingesetzt werden kann. Um besser zu verstehen, wie wichtig angewandte Mathematik für den reibungslosen Betrieb von Unternehmen und Industrieanlagen ist, machten wir uns auf den Weg, um ein Problem zu lösen, mit dem ein lokales Unternehmen konfrontiert war, indem wir unser Wissen nutzten.

So besuchten wir *Fluorocarbon Polymers* und sprachen mit dem Direktor des Unternehmens. Er erklärte uns, dass das Unternehmen verschiedene Teile und Materialien herstellt und versendet, die in der Industrie für alle möglichen Zwecke eingesetzt werden. Anschließend verfolgten wir die Produktionslinie von Anfang bis Ende und hofften, herauszufinden, wo das Unternehmen Geld und/oder Ressourcen verliert, damit wir eine Lösung finden konnten. Es erwies sich als ziemlich schwierig, Probleme in ihrem Prozess zu finden, aber schließlich beschlossen wir, den Transport der verwendeten Rohstoffe und der fertigen Produkte zu ihren Abnehmern zu optimieren.

Transportoptimierungsprobleme sind schwierig zu lösen und erfordern Kenntnisse auf Universitätsniveau. Deshalb haben wir schließlich etwas fortgeschrittene Algebra erlernt, die uns nicht nur näher an die Lösung solcher Probleme heranbrachte, sondern auch half, Probleme der linearen Algebra auf Hochschulniveau leichter zu lösen.

Im Klassenzimmer teilte uns unser Lehrer in zwei Gruppen ein: eine, die die in der Schule üblichen Methoden zur Berechnung inverser Matrizen, zur Bestimmung des Rangs einer Matrix und zur Lösung linearer Gleichungssysteme erlernen und anwenden sollte, und eine, die die fortgeschrittenen Methoden lernen sollte. Ziel war es, die verschiedenen Methoden am Ende des Kurses zu vergleichen und festzustellen, welche schneller und einfacher zu benutzen sind.

Beim Lösen der Aufgaben wurden wir ermutigt, uns gegenseitig zu helfen und schneller zu lernen. Manchmal war es tatsächlich einfacher, meinen Mitschülern Fragen zu den neuen Konzepten zu stellen, als den Lehrer zu fragen, und ihre Perspektive als Schüler ermöglichte ihnen einen gewissen nützlichen Einblick in das Problem. Der angenehmste Aspekt dieser Lernmethode war die Tatsache, dass es sich weniger wie „Lernen“ und mehr wie „Spielen“ anfühlte, da wir uns gegenseitig mit Ideen versorgten.

Am Ende stellten beide Teams ihre Methoden vor und tauschten sie aus, um ein Gefühl für die anderen Blickwinkel zu bekommen und leichter eine Schlussfolgerungen ziehen zu können. Wir waren uns alle einig, dass die fortgeschrittenen Methoden gar nicht so schwer zu verstehen waren, wie wir vorher dachten, aber es kommt wirklich darauf an, wie man sie lernt. Wenn ich auf all das zurückblicke, was wir gesehen und erreicht haben, fühle ich mich gut gerüstet für das Studium der Ingenieurwissenschaften an der Universität und für die reale Welt.

Der tatsächliche MID-DAY:

The image shows a Zoom meeting interface. The main window displays a presentation slide titled "Ce este MID-Day?". The slide content includes the "meomath" logo, the text "Co-funded by the Erasmus+ Programme of the European Union" with the EU flag, and a photograph of students in a classroom. The slide text reads: "Ce este MID-DAY? MID-Day: o zi reală a matematicii și a industriei, când elevii abordează o problemă stabilită de un partener industrial și analizată / adaptată de academicieni / cercetători și facilitată de profesori." The project number "2019-1-DE03-KA201-059604" is visible at the bottom of the slide.

The Zoom gallery view on the right shows several participants: MARINELA IANU, Nicusor Voinea, Cristian Nasta, Barni, Preda Mircea, NICOLETA MADRINSCHI, MATTEO SALIANI, and "You". The meeting title at the bottom of the Zoom window is "1:02 PM | MID-DAY: Matematica întâlnește Industria".

Below the Zoom window, a browser window is open to the website "www.fluorocarbon.co.uk". The website header reads "Fluorocarbon Polymers" and the main text says "World leading innovations in fluoropolymer processing". The website URL "www.fluorocarbon.co.uk" is also visible at the bottom of the page.

1.8: Evaluation des MID-Days: Schülerkommentare

Nach Abschluss der MID-Day-Veranstaltungen in den fünf oben genannten Ländern wurde den Teilnehmern der Veranstaltung ein Evaluationsbogen zugesandt, um ihre Zufriedenheit mit den mit dem Projekt verbundenen Aktivitäten zu ermitteln.

Das Feedback der Lernenden war insgesamt sehr positiv und ermutigend. Die Aspekte der Möglichkeit im Team zu arbeiten und Teil davon zu sein und die Zuweisung von Rollen innerhalb dieses seien sehr zufriedenstellend. Sie waren der Meinung, dass die Arbeit Spaß mache und konstruktiv sei. Alle Teilnehmer der Veranstaltungen gaben an, dass sie aus verschiedenen Gründen von der MID-Day-Aktivität profitiert hätten. Einer der Gründe war, dass sie als Mitglied eines Teams an einem realen Problem arbeiteten, dass sie ihre Kommunikations- und Präsentationsfähigkeiten weiterentwickelten, dass sie erkannten, wie kompliziert reale Probleme seien und wie die Mathematik eine Lösung für sie bieten könne. Die SchülerInnen erwähnten auch, dass die Möglichkeit der Ergebnispräsentation im Rahmen des MID-Day-Formats sich als wichtig für sie erwiesen hätte - dank dieser Gelegenheit konnten sie ihre sozialen Fähigkeiten wie Informationsaustausch und Argumentation verbessern und weiterentwickeln. Schließlich erwähnten alle Schüler, dass es für sie eine einzigartige Erfahrung war, die Welt der Industrie kennenzulernen!

Es ist wichtig zu erwähnen, dass alle Schülerinnen und Schüler aller Ländern darin übereinstimmten, dass die Probleme des MID-Tages eine viel interessantere und interaktivere Art des Lernens darstellten als die mathematischen Aufgaben, mit denen sie in ihren Schulen konfrontiert werden. Darüber hinaus gab die Mehrheit der Schüler an, dass sie durch die MID-Day-Veranstaltung etwas Neues erlernen konnten.

Als die Schülerinnen und Schüler nach ihrer Meinung zu den Vorteilen, der Praktikabilität und den Schwierigkeiten der Integration einer solchen Veranstaltung in die Lehrpläne gefragt wurden, sagten einige, dass diese Art von Aufgabenstellung im Unterricht notwendig sei und dass die Schülerinnen und Schüler durch die Arbeit in Teams mehr Vertrauen in die Mathematik gewinnen und echte Arbeitserfahrung sammeln würden. Sie glauben, dass es nicht schwierig wäre, diese Kultur in der Schule einzuführen. Andererseits äußerten einige SchülerInnen den Einwand, dass dies für sie zwar eine großartige Erfahrung gewesen sei, das Niveau der und das Interesse an Mathematik aber an den Schulen niedrig wäre und dementsprechend solche größeren Aufgabenformate die Schülerschaft weiter von der Mathematik entfremden könnten. Was die Verbesserungsvorschläge für den MID-Day betrifft, so würden die SchülerInnen in Zukunft gerne mehr Aufgabenoptionen mit unterschiedlichen Schwierigkeitsgraden sehen. Sie fanden die Aufgaben und den gesamten Prozess sehr herausfordernd und äußerten den Wunsch, dass solche Arten von Veranstaltungen und Aktivitäten nach Möglichkeit in den Lehrplan aufgenommen werden sollten.

Part C: Τα Μαθηματικά συναντούν τη Βιομηχανία σε Σχολικό Μοντέλο- Κατευθυντήριες γραμμές για εφαρμογή μίας Μέρας: Μαθηματικά στη Βιομηχανία (MID-Day) (GR)

Κεφάλαιο 1: Κατευθυντήριες γραμμές για μία Μέρα: Μαθηματικά στη Βιομηχανία (MID-Day)

Πρώτα πρέπει να αναγνωρίσετε τους χαρισματικούς μαθητές και μετά να πείσετε έναν κλάδο/βιομηχανία ότι οι ικανότητές των μαθητών για εναλλακτική σκέψη μπορεί να αξίζει για αυτούς να δοκιμάσουν να συμμετέχουν σε μια τέτοια εκδήλωση. Ταυτόχρονα, πρέπει να παρακινήσετε τους ακαδημαϊκούς και τους εκπαιδευτικούς να σας υποστηρίξουν και στη συνέχεια πρέπει να προγραμματίσετε και να σχεδιάσετε ένα MID-day (Μια μέρα: Μαθηματικά στη Βιομηχανία). Αυτό το κεφάλαιο θα παρέχει τις κατευθυντήριες γραμμές για τον προγραμματισμό και τη διεξαγωγή μιας τέτοιας εκδήλωσης.

1.1: Ορισμός και Γλωσσάρι

Εταίρο: Σε αυτές τις κατευθυντήριες γραμμές, όταν αναφερόμαστε σε εταίρους, εννοούμε έναν κλάδο έτοιμο να επενδύσει μια ή δύο μέρες συζητώντας με ακαδημαϊκούς, εκπαιδευτικούς και μαθητές προκειμένου να ορίσει και να αντιμετωπίσει ένα βιομηχανικό πρόβλημα.

Βιομηχανικό πρόβλημα: Ένα κατάλληλο ζήτημα που προβάλλεται από τον βιομηχανικό εταίρο, το οποίο είναι κατανοητό από μαθητές, εκπαιδευτικούς και ακαδημαϊκούς. Πρέπει να προσαρμοστεί και από τις δύο πλευρές, να αντιμετωπίσει ορισμένα χρήσιμα ζητήματα από την πλευρά του κλάδου και να προκαλεί δυσκολία, αλλά εφικτό από την πλευρά των μαθητών. Η κατάσταση του προβλήματος είναι το αποτέλεσμα μιας κοινής ανταλλαγής ιδεών μεταξύ του βιομηχανικού εταίρου, των ακαδημαϊκών και των εκπαιδευτικών, πριν από το MID-Day (Μια μέρα: Μαθηματικά στη Βιομηχανία). Μια λίστα πιθανών προβλημάτων παρέχεται στην ενότητα 1.4 παρακάτω.

MID-Day: Μια μέρα όπου τα Μαθηματικά συναντούν τη Βιομηχανία, όταν οι μαθητές αντιμετωπίζουν ένα πρόβλημα που έχει δημιουργηθεί από έναν βιομηχανικό εταίρο και προσαρμόζεται από τους ακαδημαϊκούς και τους εκπαιδευτικούς. Μπορεί να είναι περισσότερο ιδέες για επίλυση και σχεδιασμός, η τελειοποίηση μιας ιδέας, αντί για πλήρη λύση ενός προβλήματος.

1.2: Μεθοδολογία, Απαιτήσεις και οδηγίες για τους εκπαιδευτικούς (ο εκπαιδευτικός ως μέντορας και συνδημιουργός)

Οι ευθύνες των δασκάλων είναι να επιλέγουν και να προετοιμάζουν τους μαθητές με τρόπο που θα τους κάνει κατάλληλους για ένα καρποφόρο MID-Day. Τη μέρα του MID-Day, οι εκπαιδευτικοί θα πρέπει να διευκολύνουν την εργασία των μαθητών. Αυτό σημαίνει ότι πρώτα, ως εκπαιδευτικοί, να επιλέξετε μαθητές, είτε χρησιμοποιώντας γραπτή δοκιμασία είτε με βάση προεπιλογή από τον σύλλογο γονέων, από τους βαθμούς στο σχολείο, από συμμετοχή σε Ολυμπιάδα ή επιλογή από μαθηματικούς συνδέσμους κοκ. Επίσης πρέπει να λάβετε επίσημες συμφωνίες από γονείς και σχολεία για τη συμμετοχή όλων .

Μετά την πρώτη σας συζήτηση με τον βιομηχανικό εταίρο και τους ακαδημαϊκούς, πρέπει να αποφασίσετε για τον τύπο MID-day που θέλετε να πραγματοποιήσετε. Παρακάτω θα βρείτε μερικές προτάσεις. Θα πρέπει να δημιουργήσετε ένα μάθημα με βάση ένα σχέδιο μαθήματος που παρέχουμε, για να ανεβάσετε τους μαθητές στον κατάλληλο βαθμό ικανότητας. Αυτό συνεπάγεται επίσης την προσαρμογή των ρόλων ανάλογα με τα επίπεδα: ορισμένοι πιο προχωρημένοι ή ώριμοι μαθητές μπορούν να λειτουργήσουν ως προπονητές ή εκπαιδευτές σε νεότερους, ως οδηγοί στο πλάι παρά ως πρωταγωνιστές στη σκηνή.

Η διευκόλυνση κατά τη διάρκεια της MID-Day περιλαμβάνει την υποστήριξη των “φοβισμένων” μαθητών, την ανύψωση της αυτοπεποίθησης τους, τη διαβεβαίωσή τους ότι οι γνώσεις τους δεν είναι επαρκείς για να αρχίσουν να σκέφτονται το πρόβλημα και για να διευκολύνει την επικοινωνία, διασφαλίζοντας ότι κανείς δεν αισθάνεται ανασφαλής ή επιβάλλει την άποψή του, ότι κάθε φωνή ακούγεται, ότι κάθε μαθητής νιώθει ότι είναι αναγκαίος και εκτιμάται με τον ένα ή τον άλλο τρόπο. Η κατανομή του προβλήματος σε επιμέρους εργασίες πρέπει να προγραμματιστεί εκ των προτέρων για το σκοπό αυτό.

Ως τελευταίο βήμα θα πρέπει να αναφέρετε πώς πήγαν τα πράγματα, τηρώντας τη ρύθμιση και την αυτορρύθμιση, πώς αντέδρασε κάθε μαθητής, τι πυροδοτεί ή μειώνει το κίνητρό του, αν ένιωθε μέρος της ομάδας, τι προτάθηκε και τι δημιουργήθηκε.

Μετά το MID-Day, θα πρέπει να δώσετε ιδιαίτερη προσοχή στο πώς το υλικό που συγκεντρώθηκε κατά τη διάρκεια του MID-Day μπορεί να χρησιμοποιηθεί περαιτέρω στην καθημερινή διδασκαλία και δημιουργία γνώσης στα μαθηματικά, συσχετίζοντάς το με τη θεσμική γνώση. Περαιτέρω αναζητήστε ευκαιρίες για τους μαθητές να μοιραστούν τη δουλειά τους με άλλους σε εκθέσεις, διαγωνισμούς, φόρουμ, συλλόγους ή να οργανώσουν μια έκθεση στο σχολείο σας. Αυτό θα βοηθήσει στην μεγαλύτερη συμμετοχή των μαθητών για το επόμενο έτος. Μην ξεχάσετε να αναφέρετε ότι η συμμετοχή σε ένα MID-Day μπορεί να εκτιμηθεί από τους μαθητές στο βιογραφικό τους όταν υποβάλλουν αίτηση για υποτροφίες. Αυτή η εκδήλωση θα διευρύνει και θα γαλουχήσει το επαγγελματικό τους δίκτυο!

1.3: Μεθοδολογία, Απαιτήσεις και οδηγίες για τα πανεπιστήμια που υποστηρίζουν τις ημέρες Μαθηματικών στη Βιομηχανία (MID-Days)

Οι εταίροι του πανεπιστημίου θα πρέπει να δημιουργήσουν την ομάδα φοιτητών που θα εργαστεί για το βιομηχανικό πρόβλημα. Αυτό σημαίνει εργασία με μαθητές αλλά και εκπαίδευση των μαθητών στη διαχείριση της εργασίας ως ομάδα. Σε συνεργασία με τους βιομηχανικούς εταίρους και τους δασκάλους, πρέπει να διαμορφώσουν το πρόβλημα με τρόπο που να μπορούν να το δουλέψουν οι μαθητές.

Η κύρια ευθύνη σας είναι να προσχεδιάσετε, να οργανώσετε και να διαχειριστείτε το MID-Day και τις προπαρασκευαστικές συναντήσεις πριν από αυτό. Μια τυπική αλληλεπίδραση με εκπαιδευτικούς και μαθητές θα έχει τρεις φάσεις:

Σε μια **1η συνάντηση**: Θα συναντήσετε με άτυπο τρόπο τους μαθητές και τους εκαπιδευτικούς τους, θα τους παρακινήσετε, θα δημιουργήσετε μια ομάδα γύρω από πρωτότυπο, δημιουργικό και διασκεδαστικό μαθηματικό υλικό. Μπορείτε να απευθυνθείτε σε συμμετέχοντες σε σύνδεσμο μαθηματικών, επιστημονική έκθεση, ημέρα μαθηματικών, διαγωνισμούς μαθηματικών, διασχολικές συναντήσεις και ούτω καθεξής.

Σε μια **2η συνάντηση**: θα ανεβάσετε επιλεγμένους μαθητές ώστε να δημιουργήσουν μια ομάδα. Αυτό σημαίνει ότι θα τους προσδιορίσετε και θα τους μάθετε τα εργαλεία ώστε να εργαστούν ως ομάδα (δείτε παρακάτω).

Η **3η συνάντηση** θα αποτελείται από το MID-Day (Μέρα Μαθηματικών στη Βιομηχανία). Θα πρέπει να επιλέξετε βολικές και αρκετά μεγάλες εγκαταστάσεις για τη συνάντηση, είτε στο πανεπιστήμιο είτε στο χώρο του βιομηχανικού εταίρου, να έχετε πρόχειρα διαλείμματα για καφέ και μεσημεριανό γεύμα και ούτω καθεξής. Μια υβριδική έκδοση, με ηλεκτρονικές τηλεδιασκέψεις μπορεί να είναι κατάλληλη εάν δεν είναι δυνατή η συνάντηση με φυσική παρουσία.

Θα πρέπει να διευκολύνετε την επικοινωνία και την εργασία της ομάδας, κάνοντας τις σωστές ερωτήσεις, ώστε όλοι να αισθάνονται επαρκείς και παραγωγικοί. Προσέξτε να μην αναζητάτε απαντήσεις μόνοι σας χωρίς να βασίζεστε σε μαθητές. Βεβαιωθείτε ότι οι μαθητές είναι σχετικοί και επικεντρώνονται στην ουσία του MID-Day. Δεν θα λειτουργείτε στην εκδήλωση ως εκπαιδευτικός που διδάσκει μαθητές, θα πρέπει να θεωρούνται συνεργάτες ερευνητές.

Βασικά εργαλεία για ομαδική εργασία:

- Ξεκαθαρίστε τις διαφωνίες. Διατηρώντας ένα ασφαλές περιβάλλον, προχωρήστε σε μια καρποφόρα συζήτηση: μην ψηφίζετε, μην συμβιβάζεστε, μην επιλέγετε τυχαία, αποφύγετε τον μέσο όρο και τις διαπραγματεύσεις, αλλά προσπαθήστε για ομοφωνία.
- Ακούστε και εμπλέξτε: Μην αφήνετε κανέναν να επιβάλλει την άποψή του αλλά να διαφωνεί, μην απορρίπτετε αλλά προτείνετε αντικρούσεις.
- Μην εξατομικεύετε τις συζητήσεις, μην παίρνετε τα πράγματα προσωπικά. Ψάξτε για λύσεις.
- Αποδεχτείτε λύσεις μόνο εάν είναι κατανοητές και κοινοποιημένες από όλους.

Η παροχή μιας ελαφριάς εκπαίδευσης σε αυτά τα θέματα, τόσο για μαθητές όσο και για εκπαιδευτικούς, αυξάνει την πιθανότητα να αποκτήσουν πρωτότυπες λύσεις και μια παραγωγική ομάδα.

1.4: Προτεινόμενα παραδείγματα MID-Day (Μαθηματικά στη Βιομηχανία)

- Συζητήστε με τον βιομηχανικό σας συνεργάτη. Προβλήματα βελτιστοποίησης μπορούν να βρεθούν σε πολλές επιχειρήσεις: Είτε ο βιομηχανικός εταίρος κόβει γυαλί, ελέγχει αριθμητικά ρομπότ, διαχειρίζεται μια αποθήκη ή οργανώνει μια αλυσίδα εφοδιασμού, σίγουρα έχουν ένα βασικό ανοιχτό πρόβλημα βελτιστοποίησης που πρέπει να διευκρινιστεί εάν δεν λυθεί.
- Μια Μελέτη Στατιστικής Έρευνας θα ενδιαφέρει τον βιομηχανικό συνεργάτη σας, μπορεί να θέλει να μάθει καλύτερα τις ανάγκες των πελατών ή των εργαζομένων του, και ένα ερευνητικό έργο αγοράς ή ανθρώπινου δυναμικού είναι σχετικά εύκολο στη δημιουργία, διασκεδαστικό και δημιουργικό στο σχεδιασμό και μπορεί να οδηγήσει σε απροσδόκητα αποτελέσματα όταν τα δούμε από μια ασυνήθιστη σκοπιά: «πώς να μεταμορφώσουμε αυτόν τον ζοφερό ανοιχτό χώρο στο πλαίσιο του Covid;». Ένα σχέδιο μαθήματος στα στατιστικά στοιχεία και τη μεθοδολογία έρευνας είναι απαραίτητο πριν ξεκινήσετε πραγματικά τη δουλειά.
- Μια Επιστημονική Διαδρομή στη βιομηχανική τοποθεσία είναι ένας πολύ καλός τρόπος για τη βιομηχανία να προωθήσει τη δουλειά της κατά τη διάρκεια των επισκέψεων στην τάξη, και για τους μαθητές να γίνουν κριτές και να βλέπουν το περιβάλλον τους με επιστημονικό μάτι: «πόσο μεγάλη είναι αυτή η δεξαμενή; πόσο καιρό θα πάρει για να το γεμίσει;» Το σχέδιο μαθήματος θα πρέπει να επικεντρώνεται στη μοντελοποίηση και τη μεθοδολογία μέτρησης, ανακαλύπτοντας για παράδειγμα ότι το σώμα σας είναι ένα αρκετά ακριβές εργαλείο για καθημερινές εκτιμήσεις ποσοτήτων.
- Εάν δεν έχετε άμεση επαφή με έναν βιομηχανικό εταίρο, μην ξεχνάτε την εκπαιδευτική βιομηχανία και ζητήστε από τους μαθητές να δημιουργήσουν παιδαγωγικά παιχνίδια για εσάς: αφήστε τους να εφεύρουν νέους τρόπους διδασκαλίας παιχνιδιοποιώντας μέρος του περιεχομένου σας. Μάθετε από αυτούς! Το σχέδιο μαθήματος πρέπει να διδάσκει τη νευρωνική βάση της γνώσης, την παιχνιδιοποίηση, τον καθορισμό στόχων και υποστόχων, τον καθορισμό προκλήσεων και επιπέδων, τρόπους συνεργασίας, βοηθώντας τους μαθητές να κατανοήσουν καλύτερα τη δική τους γνωστική διαδικασία. Και ποτέ δεν καταλαβαίνεις μια έννοια καλύτερα από το να πρέπει να την εξηγήσεις, ειδικά σε έναν υπολογιστή!

Μόλις οριστεί το κύριο θέμα, μπορείτε να σχεδιάσετε τη πορεία του μαθήματος για να φέρετε τους μαθητές από τη θέση τους στις απαιτούμενες ικανότητες σχετικά με τις εργασίες που εκτελούν. Δηλαδή: πρέπει να διεξάγετε το μάθημα σε μια εισαγωγή στο έργο. Αυτή η πρώτη μέρα στο σχολείο πρέπει να είναι χαλαρή και χαρούμενη, οι μαθητές να εργάζονται σε ομάδες και να γνωρίζονται μεταξύ τους.

Μόλις το πρόβλημα διορθωθεί και καθαριστεί με τον συνεργάτη, θα πρέπει να το στείλετε στους μαθητές εκ των προτέρων ώστε να έχουν τη δυνατότητα να αντιδράσουν, να αλληλεπιδράσουν, και να κάνουν διευκρινιστικές ερωτήσεις.

Κατά τη διάρκεια της μεγάλης ημέρας του MID-Day, οι μαθητές επισκέπτονται τον βιομηχανικό χώρο ή τουλάχιστον συζητούν με εκπροσώπους του κλάδου και εργάζονται μαζί στο έργο τους το πρωί.

Ακολουθεί ένα πιθανό πρόγραμμα για το MID-Day:

- 9-9:30: Καλωσόρισμα, συζητήσεις
- 9:30-10:30: Δέσιμο της ομάδας με μία ομαδική δραστηριότητα όπως η «χαμένος στο φεγγάρι» ή παρόμοια: <https://netmind.net/play-the-moon-landing-exercise/>
- 10:30-10:45: Διάλειμμα
- 10:45-11:45: Συζήτηση σε υποομάδες, σχετικά με το πρόβλημα του MID-Day, κατανόηση του θέματος, πρώτα κομμάτια σκέψεων...
- 11:45-12:15: Πρώτη σύνθεση για πιθανές στρατηγικές αντιμετώπισης των ζητημάτων, ορίζοντας συγκεκριμένες εργασίες.
- 12:15-13:45: Διάλειμμα για μεσημεριανό γεύμα
- 13:45-15:45: Εργαστείτε σε υποομάδες σε συγκεκριμένες στρατηγικές και εργασίες.
- 15:45-16: Διάλειμμα
- 16-16:30: Προετοιμασία της σύνθεσης και της παρουσίασης [Οι μαθητές]
- 16:30-17:15: Παρουσίαση ενώπιον των βιομηχανικών εταίρων, συζήτηση

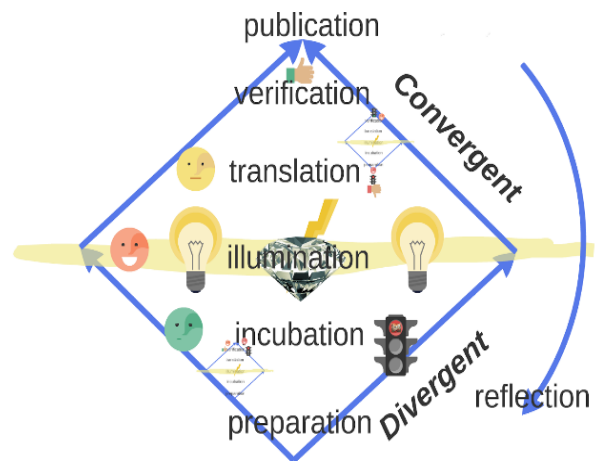
1.5: Αποτελέσματα αξιολόγησης, Τι να κάνετε και τι να μην κάνετε

Κατά τη διάρκεια του MID-Day:

Προσέξτε ότι κάθε μαθητής πρέπει να ανήκει σε μια μικρή ομάδα, το πολύ 8 μαθητές, η οποία θα σχηματιστεί στις προπαρασκευαστικές δραστηριότητες, για παράδειγμα με μεγαλύτερους μαθητές να επιβλέπουν την ομάδα ως προπονητές, κάνοντας τους πάντες να αισθάνονται ότι είναι απαραίτητοι και ότι ανήκουν. Ιδιαίτερη προσοχή πρέπει να δοθεί στους ανασφαλείς μαθητές κάνοντας κατάλληλες ερωτήσεις, αναδεικνύοντας και αναπτύσσοντας τις ιδιαίτερες ικανότητες κάθε μαθητή, αυξάνοντας την αυτοπεποίθησή τους. Το βιομηχανικό πρόβλημα αναλύεται με κάθε προοπτική κατά νου, έτσι ώστε να είναι δυνατός ο επιμερισμός των εργασιών μεταξύ όλων των μελών της ομάδας, ενισχύοντας τη συνεργασία όλων των μελών της.

Η αξιολόγηση πρέπει να επικεντρωθεί σε αυτές τις πτυχές: ότι κάθε μαθητής ένιωθε ότι λαμβάνεται υπόψη ως μεμονωμένος στοχαστής και δημιουργός. Φυσικά, δεν μπορούν όλες οι ιδέες να οδηγήσουν σε ένα επιτυχημένο προϊόν, ίσως δεν θα προκύψει παραγωγή από το MID-Day και δεν θα πρέπει να θεωρηθεί ως αποτυχία, αλλά θα πρέπει να προβληθούν οι δυνατότητες, η δημιουργικότητα και η παραγωγικότητα των ιδεών τους. Εάν χρειάζεστε αριθμούς ή θέλετε να εκχωρήσετε μεμονωμένα τρόπαια, μπορείτε να αντικειμενοποιήσετε την αξιολόγηση της δημιουργικότητας της διαδικασίας ιδεασμού που έλαβε χώρα στις τέσσερις διαστάσεις που περιγράφονται από τον Guilford: Ρευστότητα, Ευελιξία, Επεξεργασία και Πρωτοτυπία:

- Ρευστότητα είναι η ικανότητα να παράγεις πολλά, να επεξεργαστείς πολλά παραδείγματα.
- Ευελιξία είναι η ικανότητα να αλλάζεις μια στρατηγική όταν είσαι κολλημένος, η ηρεμία και η προσαρμογή είναι ξεκάθαρα οι λέξεις-κλειδιά εδώ.
- Η επεξεργασία σκάβει σε βάθος μια ερώτηση, αναζητά τις λεπτές λεπτομέρειες, είναι εξαντλητική στην αναζήτησή της.
- Η πρωτοτυπία είναι το είδος της δημιουργικότητας που αναγνωρίζεις όταν τη βλέπεις: δεν έχεις ξαναδεί κάτι τέτοιο, είναι νέο και προκλητικό.



Εφόσον μιλάμε για ιδεασμό, να γνωρίζετε ότι δεν είμαστε στο κεφάλι των μαθητών, για να είμαστε πλήρως δημιουργικοί, μια ιδέα πρέπει να περάσει από ολόκληρο τον κύκλο της δημιουργικότητας με τις δύο αποκλίνουσες και συγκλίνουσες φάσεις: από την προετοιμασία στην επώαση, μετά φώτιση (η στιγμή Εύρηκα!), αλλά αυτή η ιδέα πρέπει να βγει από το μυαλό του μαθητή και να μεταφραστεί σε λέξεις, σκίτσα και ούτω καθεξής, για να επαληθευτεί και μετά να δημοσιευτεί, να προτείνει στους συναδέλφους να δουν τι πιστεύουν γι' αυτό, είτε το επικυρώνουν είτε όχι. Και στο τέλος αυτή η ιδέα μπορεί να αντικατοπτριστεί και να ενσωματωθεί σε έναν μεγαλύτερο κύκλο ιδεών προκειμένου να αντιμετωπιστεί το πρόβλημα που διακυβεύεται.

Δεν πρέπει να μπερδεύουμε τη δημιουργικότητα με την αποκλίνουσα σκέψη: μια φάση καταιγισμού ιδεών είναι εντάξει, αλλά κάποια στιγμή πρέπει να απομακρυνθεί από την αφηρημένη και να συγκλίνει σε μια πραγματική εφαρμόσιμη λύση, και υπάρχει ανάγκη για τόση δημιουργικότητα σε αυτή τη συγκλίνουσα φάση. Ομοίως, η στιγμή του φωτισμού Εύρηκα! δεν πρέπει να υπερεκτιμηθεί, η ποίηση και η βιομηχανία μπορούν να αναμειχθούν μόνο σε ένα σημείο, αυτό που φαίνεται σαν μια πρωτότυπη ιδέα πρέπει να μεταφραστεί σε πραγματικές

λύσεις. Αυτά τα λεπτά σημεία πρέπει να ληφθούν υπόψη από την ομάδα προκειμένου να καλλιεργηθεί ένας παραγωγικός τρόπος σκέψης μεταξύ των μαθητών.

Μετά το MID-Day:

Όλα τα αποτελέσματα δεν θα εμφανιστούν αμέσως κατά τη διάρκεια του MID-Day. Οι μαθητές θα επιστρέψουν στο σπίτι με πολλές παρατηρήσεις, ιδέες και σχέδια. Στην τελευταία φάση, η εργασία αυτή μπορεί να επεξεργαστεί εκ νέου και να παρουσιαστεί ξανά στον βιομηχανικό εταίρο. Είναι σημαντικό να οργανωθεί ένας τρόπος για να ανταλλάσσουν συζητήσεις με άλλους. Αυτό μπορεί να χρησιμοποιηθεί περαιτέρω, για σχολικές εκθέσεις, φόρουμ, εβδομάδες ένταξης, διαγωνισμούς και ούτω καθεξής, προκειμένου να φανεί τι έχει επιτευχθεί.

1.6: Γιατί η βιομηχανία πρέπει να εμπλακεί, οφέλη για τη βιομηχανία

Αυτό το σημείο του ενδιαφέροντος του βιομηχανικού εταίρου πρέπει να ξεκαθαριστεί εκ των προτέρων, αλλά είναι καλό να το επανεξετάσουμε στο τέλος της ημέρας και να εξετάσουμε τη λίστα ελέγχου: Έχουν επωφεληθεί όλα τα μέρη από το MID-Day; Έχει η εναλλακτική σκέψη πραγματικά ξεμπλοκάρει μια κατάσταση δημιουργικότητας ή απλά η βιομηχανία ήταν σε θέση να δείξει την επιχείρησή της σε ξένους. Τα οφέλη είναι πραγματικά διαφορετικά ανάλογα με τον τύπο του MID-Day που διεξήχθη. Για να εξετάσουμε τα γενικά παραδείγματα που δώσαμε:

- Έχει η Έρευνα Στατιστικής καταφέρει να κάνει εμφανή στοιχεία που δεν είχαν εντοπιστεί στο παρελθόν ή ήταν απλώς ένας τρόπος επικοινωνίας, είτε εσωτερικά είτε με πιθανούς πελάτες; Το να επιτρέπεται στους πελάτες ή στους καλύτερους υπαλλήλους να εκφράζουν τις απόψεις τους, να βλέπουν ότι λαμβάνονται υπόψη είναι ήδη κάτι που μπορεί να εκτιμηθεί.
- Είναι το Επιστημονικό Μονοπάτι διασκεδαστικό και ενημερωτικό; Μπορεί να χρησιμοποιηθεί με πιθανούς πελάτες, με μαθήματα; Ανοίγει τα μάτια των συμμετεχόντων στις προκλήσεις που αντιμετωπίζει ο κλάδος; Σε περίπτωση τουριστικής επίσκεψης σε χώρο, ανακαλύπτει ο συνεργάτης τον χώρο με νέο μάτι; Θα εξυπηρετήσει το σκοπό του επιστημονικού τουρισμού;
- Όσο για την εκπαιδευτική βιομηχανία, είχε η οργάνωση των μαθητών ενδιαφέροντα, ενημερωτικά και διασκεδαστικά παιδαγωγικά παιχνίδια; Έμαθαν κάτι από το σχεδιασμό του παιδαγωγικού παιχνιδιού;
- Ο κλάδος θα έχει την ευκαιρία να έρθει σε επαφή με ταλαντούχους φοιτητές και ενδεχομένως να συνεργαστεί μαζί τους στο μέλλον.

1.7: Εφαρμογή του MID – Ημέρες σε διάφορες Χώρες:

Η εκδήλωση MID- Day εφαρμόστηκε πιλοτικά και υλοποιήθηκε σε πέντε ευρωπαϊκές χώρες. Αυτές είναι: Κύπρος, Γαλλία, Γερμανία, Πολωνία και Ρουμανία. Η ιδέα πίσω από την εκδήλωση MID – Day είναι ότι η εφαρμογή των μαθηματικών στην πραγματική ζωή είναι απολύτως απαραίτητη για κάθε πρόγραμμα σπουδών στις μέρες μας και ακόμη περισσότερο για τους χαρισματικούς μαθητές που μπορούν να συμβάλουν στην καινοτομία που θα μπορούσε να κάνει την ανθρώπινη ζωή καλύτερη και ταυτόχρονα να προωθήσει την επιχειρηματικότητα στα σχολικά προγράμματα σπουδών. Η εκδήλωση Mathematics meets industry έχει σχεδιαστεί για να προσφέρει στους μαθητές με πρόκληση άλυτα προβλήματα της βιομηχανίας στην πραγματική ζωή με στόχο να εργαστούν είτε σε ομάδες είτε μεμονωμένα και να χρησιμοποιήσουν τα μαθηματικά για να προτείνουν μια σειρά πιθανών λύσεων στα προβλήματα που δίνονται. Επιπλέον, οι μαθητές είχαν την ευκαιρία να συνεργαστούν μεταξύ τους και να συνεργαστούν για την επίτευξη ενός κοινού στόχου, να συναντήσουν ερευνητές της τριτοβάθμιας εκπαίδευσης καθώς και ανθρώπους της βιομηχανίας και για τους εκπαιδευτικούς δευτεροβάθμιας εκπαίδευσης να δουν, να πάρουν ιδέες και να εμπνευστούν για το πώς τέτοια προβλήματα θα μπορούσαν να χρησιμοποιηθούν ως μέρος της τακτικής διδασκαλίας τους.

Η προετοιμασία και η υλοποίηση της εκδήλωσης διέφεραν σε κάθε χώρα. Παρουσιάστηκαν διάφορα προβλήματα της βιομηχανίας, τα οποία οι μαθητές κλήθηκαν να λύσουν. Ως εκ τούτου, παρακάτω θα αναλύσουμε την προετοιμασία και την υλοποίηση κάθε εκδήλωσης MID-day ανά χώρα. Με αυτόν τον τρόπο μπορούν να εξαχθούν παραδείγματα και προτάσεις σχετικά με τη μεθοδολογία της MID-day.

Στην περίπτωση της Κύπρου

Δράσεις προετοιμασίας: Η προετοιμασία αυτής της εκδήλωσης ξεκίνησε δύο μήνες νωρίτερα, όταν εστάλη μια επιστολή πρόσκλησης/κινήτρου σε μαθητές που απολαμβάνουν μαθηματικά και μπορεί να ενδιαφέρονταν για μια τέτοια δραστηριότητα. Εν τω μεταξύ, η ομάδα εκπαιδευτικών INNOMATH του Heritage Private School επικοινωνήσε με τις τοπικές επιχειρήσεις, μετά από τη διοργάνωση ενός ομίλου εστίασης, και τους ζήτησε να συμμετάσχουν σε αυτήν την εκδήλωση παρέχοντας στους μαθητές ένα πραγματικό πρόβλημα βιομηχανίας. Δύο εταιρείες παρείχαν ένα πρόβλημα η καθεμία για τους σκοπούς της MID-Day. Το πρόβλημα A δόθηκε από τα V-ships που είναι μία από τις μεγαλύτερες ναυτιλιακές εταιρείες στον κόσμο. Το πρόβλημα περιέγραψε μια συγκεκριμένη περίπτωση ενός δεξαμενόπλοιου που εμπορεύεται μεταξύ Σιγκαπούρης και Βραζιλίας, το οποίο πρέπει να εφοδιαστεί με προμήθειες προκειμένου να υπάρχουν ανά πάσα στιγμή προμήθειες. Σε αυτό το έργο, οι μαθητές έπρεπε να δράσουν προς όφελος του παρόχου τροφοδοσίας σε μια προσπάθεια να διαμορφώσει το πλάνο από οικονομικής αποδοτικότητας σχέδιο αναπλήρωσης προμηθειών, διασφαλίζοντας ότι τηρούνται οι συμφωνηθείσες τιμές, το πλοίο θα έχει επαρκές απόθεμα επί του πλοίου ανά πάσα στιγμή κατά τη διάρκεια του ταξιδιού και ταυτόχρονα θα έπρεπε να μεγιστοποιήσει το κέρδος για την εταιρεία παροχής υπηρεσιών τροφοδοσίας. Το Πρόβλημα B δόθηκε από μια αρχιτεκτονική εταιρεία και ζητήθηκε από τους φοιτητές να πάρουν τη θέση ενός αρχιτέκτονα, να κατανοήσουν τη διαδικασία και να βρουν την καλύτερη δυνατή λύση για την εξυπηρέτηση ενός πελάτη με βάση τις απαιτήσεις του σπιτιού του, τα υλικά που θα χρησιμοποιηθούν και τον διαθέσιμο προϋπολογισμό.

Οι μαθητές που προετοιμάζαν τις MID-DAY παρουσιάσεις τους: Πολλοί μαθητές που αγαπούν τα μαθηματικά, επέλεξαν να συμμετάσχουν στη MID-DAY δραστηριότητα. Ήταν μια ευκαιρία για αυτούς να σκεφτούν δημιουργικά και έξω από το κουτί, είτε μεμονωμένα είτε σε ομάδες, προκειμένου να βρουν πιθανές λύσεις στα προβλήματα που τους δίνονται σε μια εκτός της τάξης δραστηριότητα. Λόγω των περιορισμών του Covid19 και των μαθητών που συμμετείχαν προερχόμενοι από διαφορετικές πόλεις, διοργανώθηκαν δύο διαδικτυακές συναντήσεις με τους εκπαιδευτικούς του Heritage, τους συμμετέχοντες μαθητές, τους εκπροσώπους της εταιρείας και τη Μαθηματική Εταιρεία. Οι διαδικτυακές συναντήσεις πραγματοποιήθηκαν στις αρχές και στα μέσα Ιουλίου 2021. Στόχος των συναντήσεων ήταν οι μαθητές να δημιουργήσουν τις ομάδες τους, να εξοικειωθούν με τα προβλήματα, να λάβουν επιπλέον εξηγήσεις και διευκρινίσεις από τους εκπαιδευτικούς καθώς και από τους εκπροσώπους της εταιρείας. Δόθηκαν επίσης γενικές οδηγίες για την ομαλή λειτουργία του MID-Day και τις απαιτήσεις για τις παρουσιάσεις των μαθητών. Μετά την πρώτη συνάντηση οι μαθητές είχαν μερικές μέρες για να αποφασίσουν για το πρόβλημα της προτίμησής τους και άρχισαν να εργάζονται πάνω σε αυτό. Το πρόβλημα A ήταν πιο δημοφιλές καθώς επιλέχθηκε από όλους τους μαθητές.

Η πραγματική MID-DAY: Την Τετάρτη 28 Ιουλίου 2021 πραγματοποιήθηκε στον Αγρό η εκδήλωση παρουσία φοιτητών, γονέων, εκπαιδευτικών, ερευνητών τριτοβάθμιας εκπαίδευσης και συνεργατών του έργου. Οι μαθητές παρουσίασαν τις πιθανές λύσεις τους και στο τέλος κάθε παρουσίασης ο εκπρόσωπος των V-ships, κ. Λουκάς Χρυσόχοι, έδινε τα σχόλιά του για τις λύσεις που πρότειναν οι μαθητές. Επίσης, όλοι οι άλλοι συμμετέχοντες ήταν ελεύθεροι να κάνουν ερωτήσεις στους μαθητές. Όλες οι λύσεις που ελήφθησαν από τους μαθητές ήταν διαφορετικές και αυτό ξεκίνησε μια γόνιμη συζήτηση μεταξύ των συμμετεχόντων μαθητών και του κοινού. Στο τέλος, ο εκπρόσωπος του κλάδου εξέφρασε τον ενθουσιασμό του για τη σκληρή δουλειά των μαθητών, τις εξαιρετικές δεξιότητες παρουσίασης και τον αναλυτικό τρόπο παρουσίασης της λύσης τους. **Ανέφερε ότι ήταν μια μεγάλη εμπειρία για αυτόν να συμμετάσχει σε αυτή την εκδήλωση και να συναντήσει τέτοιους μαθητές που έκαναν εξαιρετική δουλειά. Επιπλέον, εξέφρασε το ενδιαφέρον του να συνεργαστεί μαζί τους μια μέρα.**

Η MID-DAY στην πράξη:



Η περίπτωση της Γαλλίας

Οι μαθητές που συμμετέχουν στο MID-Day φορούν δύο καπέλα, γνωρίζουν ότι δεν συμμετέχουν μόνο αλλά και δοκιμάζουν το παιδαγωγικό καινοτόμο σχέδιο. Οι μαθητές ξεκίνησαν με ένα δημιουργικό τρόπο σκέψης, όπου τα μαθηματικά θεωρήθηκαν ως ένα συγκεκριμένο εργαλείο για την επίλυση πραγματικών προβλημάτων οικονομικής αξίας.

Οι μαθητές αφέθηκαν να εγκατασταθούν σε ομάδες μέσω icebreakers και στη συνέχεια εγκατέστησαν την εφαρμογή MathCityMap και έτρεξαν το τοπικό μαθηματικό ίχνος.

Πέρασαν το μεγαλύτερο μέρος της ημέρας στο πεδίο, στην αναγεννησιακή συνοικία της UNESCO, ακούγοντας τις ιστορίες που έλεγε ο επαγγελματίας οδηγός, παίρνοντας σημειώσεις, στήνοντας πράγματα σε μια ομάδα, δοκιμάζοντας ιδέες, επιτρέποντας σε άλλα μέλη της ομάδας να ελέγξουν τη σκοπιμότητα και τη διασκέδαση των εργασιών... Πίσω στο πανεπιστήμιο, οι εργασίες κωδικοποιήθηκαν στο σύστημα MathCityMap.

Το κύριο θέμα ήταν να απαντηθούν οι ανάγκες του εκπροσώπου της τουριστικής βιομηχανίας: οι μαθητές συζήτησαν τους καλύτερους τρόπους αντιμετώπισης διαφορετικών στόχων, οικογενειών, τάξεων, ομάδων φίλων. Η κατανόηση των περιορισμών του επαγγελματία οδηγού, η έγχυση αρκετά διασκεδαστικών και εντυπωσιακών πληροφοριών μέσα σε επιστημονικά ερωτήματα είναι ο κύριος στόχος της μαθηματικής διαδρομής που παρήχθη.

Το δεύτερο μέρος του MID-Day ήταν η ανατροφοδότηση προς τον βιομηχανικό εταίρο. Οι μαθητές είχαν την ευκαιρία να συνεργαστούν για να στήσουν την ομιλία τους.

Πραγματοποιήθηκε κατά τη διάρκεια της διακρατικής συνάντησης έργου που πραγματοποιήθηκε στη Λυών στα μέσα Δεκεμβρίου 2021 στην πανεπιστημιούπολη του UCBL. Την επομένη της διαδρομής οι μαθητές δικαιολόγησαν τις επιλογές που έκαναν μπροστά στην κριτική επιτροπή που αποτελούνταν από τον εκπρόσωπο του κλάδου και τους διεθνείς εταίρους του έργου InnoMath.



Στην περίπτωση της Γερμανίας

Δράσεις προετοιμασίας: Η προετοιμασία της εκδήλωσης MID-day ξεκίνησε 6 μήνες νωρίτερα με μια συνάντηση με εταίρους του κλάδου που συζητούσαν πιθανά καθήκοντα. Η συνεδρίαση της Ομάδας Εστίασης, η οποία αποτελούνταν από εκπαιδευτικούς, επιστήμονες και βιομηχανικούς εταίρους, συνάντησε και αξιολόγησε τις ιδέες και το προτεινόμενο χρονοδιάγραμμα για το MID-DAY. Αργότερα, σχολεία με έμφαση στα μαθηματικά και τις φυσικές επιστήμες κλήθηκαν να συμμετάσχουν στην εκδήλωση. Μαζί με το Stiftung Planetarium Berlin, αποφασίστηκε να τεθεί το πρόβλημα στους μαθητές για το πώς οι αλλαγές στις εποχές - κλιματική αλλαγή - μπορούν να γίνουν στατιστικά ορατές και πώς μπορούν να γίνουν προβλέψεις. Ειδικότερα, τα μοντέλα που χρησιμοποιήθηκαν έπρεπε να αξιολογηθούν και να αντανακλώνται.

Οι μαθητές που προετοιμάζαν τις MID-DAY παρουσιάσεις τους: 14 τάξεις (420 μαθητές) σχολείων με έμφαση στα μαθηματικά και τις φυσικές επιστήμες συμμετείχαν στη MID-DAY δραστηριότητα. Λόγω των περιορισμών του Covid19 οι μαθητές έπρεπε να συναντηθούν στα σχολεία τους και δεν ήταν σε θέση να συναντήσουν μαθητές από άλλα σχολεία για την προετοιμασία της παρουσίασης. Οι προετοιμασίες πραγματοποιήθηκαν το Σεπτέμβριο και δόθηκε στους μαθητές υλικό για να εξοικειωθούν με τα προβλήματα, τα οποία προετοιμάστηκαν από τους ερευνητές και τους εταίρους του κλάδου. Οι εκπαιδευτικοί στήριξαν τους μαθητές και τους επέβλεπαν. Δόθηκαν επίσης γενικές οδηγίες για την ομαλή λειτουργία του MID-DAY και τις απαιτήσεις για τις παρουσιάσεις των μαθητών. Μετά την πρώτη συνάντηση οι μαθητές είχαν μερικές μέρες για να αποφασίσουν για την εστίαση της λύσης και τις παρουσιάσεις τους, η οποία στη συνέχεια συζητήθηκε στην τάξη. Οι μαθητές αποφάσισαν πώς να οργανώσουν τη δουλειά τους και πώς να μοιραστούν το φόρτο εργασίας μέσα στις ομάδες. Οι μαθητές είχαν επίσης την ευκαιρία να ζητήσουν περισσότερη βοήθεια και διευκρινίσεις που θα μπορούσαν να τους βοηθήσουν να εργαστούν μέσω της προτεινόμενης λύσης τους.



Στην περίπτωση της Πολωνίας

Δράσεις προετοιμασίας: Οι προετοιμασίες ξεκίνησαν ένα μήνα νωρίτερα. Οι μαθητές έπρεπε να εξοικειωθούν με το έργο που παρουσίασε το τουριστικό γραφείο. Συνίστατο στην κατασκευή ενός αλγορίθμου που επιτρέπει τη δημιουργία ενός σχεδίου ταξιδιού για τους μαθητές που επισκέπτονται την Κρακοβία. Το σχέδιο αυτό έπρεπε να λάβει υπόψη ορισμένες προϋποθέσεις, όπως: προϋπολογισμός, διάρκεια, ηλικία συμμετεχόντων, επιλογή προαιρετικών αξιοθέατων και άλλες.

Οι μαθητές προετοίμασαν τις MID-DAY παρουσιάσεις τους: Καθώς στην εκδήλωση συμμετείχαν ομάδες μαθητών από τα ίδια μαθήματα, οι προετοιμασίες πραγματοποιήθηκαν στη στατική έκδοση. Το έργο τους συνίστατο στη συλλογή σχετικών δεδομένων, όπως: τιμές εισιτηρίων, απόσταση μεταξύ σημείων, τιμές ενοικίασης πούλμαν, τιμές δημόσιων συγκοινωνιών και άλλα. Στη δεύτερη φάση των προετοιμασιών, το καθήκον τους ήταν να δημιουργήσουν ένα πρόγραμμα για να προσομοιώσουν το κόστος της εκδήλωσης. Τόσο οι παραδοσιακές μέθοδοι όσο και οι σύγχρονες τεχνολογίες πληροφοριών χρησιμοποιήθηκαν για την προετοιμασία. Ορισμένες από τις εργασίες έγιναν επίσης εξ αποστάσεως. Στην τελική φάση, πριν από την εκδήλωση, τα μέλη της ομάδας επικεντρώθηκαν στην παρουσίαση των επιτευγμάτων τους, δηλαδή στην προετοιμασία και την εκπαίδευση των ομιλιών τους.

Το πραγματικό MID-DAY: Η εκδήλωση χωρίστηκε σε τρία μέρη: Στο πρώτο μέρος, παρουσιάστηκε το έργο INNOMATH, οι στόχοι, οι υποθέσεις και τα αποτελέσματά του που αναπτύχθηκαν κατά τη διάρκειά του. Το δεύτερο μέρος συνίστατο στην παρουσίαση των έργων των μαθητών. Όλες οι ομάδες παρουσίασαν τα αποτελέσματα των προετοιμασιών τους. Μετά από κάθε ένα από αυτά, οι ειδικοί και οι προσκεκλημένοι είχαν την ευκαιρία να εκφράσουν τις απόψεις τους για τις προετοιμασμένες παρουσιάσεις. Το τρίτο μέρος ήταν μια συζήτηση για θέματα που σχετίζονται με το έργο και τον αντίκτυπό του στη σύγχρονη προσέγγιση της διδασκαλίας.



Στην περίπτωση της Ρουμανίας:

Σημείωση: Αυτό το παράδειγμα του MID-Day είναι γραμμένο από την οπτική των μαθητών

Κατά τη διάρκεια του έργου INNOMATH βιώσαμε πολλά νέα πράγματα που μας έδειξαν πώς είναι η ζωή πέρα από το λύκειο και πώς τα μαθηματικά μπορούν να χρησιμοποιηθούν σε πραγματικές καταστάσεις. Για να κατανοήσουμε καλύτερα πόσο πεμπτουσία είναι τα εφαρμοσμένα μαθηματικά για την ομαλή λειτουργία των επιχειρήσεων και των βιομηχανικών εγκαταστάσεων, ξεκινάμε ένα ταξίδι για να προσπαθήσουμε να λύσουμε ένα ζήτημα που αντιμετώπιζε μια τοπική εταιρεία, αξιοποιώντας τις γνώσεις μας.

Έτσι, επισκεφθήκαμε την Fluorocarbon Polymers και μιλήσαμε με τον διευθυντή της. Μας εξήγησε ότι παράγουν και μεταφέρουν διάφορα κομμάτια και υλικά που εξυπηρετούν κάθε είδους σκοπό στη βιομηχανία. Στη συνέχεια προχωρήσαμε στην παρακολούθηση της γραμμής παραγωγής από την αρχή μέχρι το τέλος, ελπίζοντας να εντοπίσουμε τρόπους με τους οποίους η εταιρεία χάνει χρήματα ή / και πόρους, ώστε να μπορέσουμε να βρούμε μια λύση. Αποδείχθηκε αρκετά δύσκολο να βρεθούν ζητήματα στη διαδικασία τους, αλλά στο τέλος, αποφασίσαμε να προσπαθήσουμε να βελτιστοποιήσουμε τη μεταφορά των πρώτων υλών που χρησιμοποιούν και των τελικών προϊόντων στους αγοραστές τους.

Τα προβλήματα βελτιστοποίησης των μεταφορών είναι δύσκολο να επιλυθούν και απαιτούν πανεπιστημιακές δεξιότητες, επομένως αυτό που καταλήξαμε να κάνουμε είναι να μάθουμε κάποια προηγμένα άλγεβρα που μας έφερε μόνο πιο κοντά στο να μπορέσουμε να λύσουμε τέτοια προβλήματα, βοηθώντας μας επίσης να λύσουμε πιο εύκολα γραμμικά προβλήματα άλγεβρας επιπέδου λυκείου.

Στην τάξη ο εκπαιδευτικός μας μας χώρισε σε δύο ομάδες: μία που θα μάθαινε και θα χρησιμοποιούσε τις μεθόδους που συνήθως διδάσκονται στο γυμνάσιο για την πληροφορική αντίστροφων πινάκων, τον προσδιορισμό της τάξης ενός πίνακα και την επίλυση συστημάτων γραμμικών εξισώσεων και ένα που θα μάθαινε τις προηγμένες μεθόδους. Ο στόχος ήταν να συγκριθεί με τις διάφορες μεθόδους στο τέλος του μαθήματος και να καθοριστεί ποιες είναι ταχύτερες και ευκολότερες στη χρήση.

Καθώς λύναμε ασκήσεις επίλυσης, μας ενθάρρυναν να βοηθήσουμε ο ένας τον άλλον για να κατανοήσουμε και να μάθουμε πιο γρήγορα. Πράγματι, μερικές φορές ήταν ευκολότερο να κάνω στους συμμαθητές μου ερωτήσεις σχετικά με τις νέες έννοιες παρά να ρωτήσω τον εκπαιδευτικό και η οπτική τους ως μαθητές τους επέτρεψε να έχουν μια χρήσιμη εικόνα του προβλήματος. Η πιο ευχάριστη πτυχή αυτής της μεθόδου μάθησης ήταν το γεγονός ότι την αντιληφθήκαμε λιγότερο σαν μάθηση και περισσότερο σαν παιχνίδι καθώς ανταλλάζαμε ιδέες μεταξύ μας.

Στο τέλος, και οι δύο ομάδες παρουσίασαν και αντάλλαξαν τις μεθόδους που είχαν χρησιμοποιήσει για να πάρουν μια αίσθηση για το άλλο μισό αυτής της ιστορίας και να είναι σε θέση να βγάλουν συμπεράσματα πιο εύκολα. Όλοι συμφωνήσαμε ότι οι προηγμένες μέθοδοι δεν ήταν τόσο δύσκολο να κατανοηθούν όσο νομίζαμε προηγουμένως, αλλά έχει πραγματικά σημασία πώς τις μαθαίνεις. Κοιτάζοντας πίσω σε όλα όσα έχουμε δει και επιτύχει, αισθανόμαστε καλά που επιλέξαμε να σπουδάσουμε μηχανική για σπουδές και έτσι είμαστε καλύτερα προετοιμασμένοι να αντιμετωπίσουμε τον πραγματικό κόσμο.

Η MID-DAY στην πράξη:

The image shows a Zoom meeting interface. The main window displays a presentation slide titled "Ce este MID-Day?". The slide features the "meomath" logo, the text "Co-funded by the Erasmus+ Programme of the European Union" with the EU flag, and a photograph of students in a classroom. The slide text reads: "Ce este MID-DAY? MID-Day: o zi reală a matematicii și a industriei, când elevii abordează o problemă stabilită de un partener industrial și analizată / adaptată de academicieni / cercetători și facilitată de profesori." The project number "2019-1-DE03-KA201-059604" is visible at the bottom. A video feed of a woman, identified as "MARINELA JANA", is visible in the top right corner.

Below the Zoom window, a browser window shows a Google Meet link: <https://meet.google.com/et-tdt-dqj?authuser=3>.

The bottom part of the image shows a Zoom meeting grid. The main window displays a presentation slide titled "Fluorocarbon Polymers" with the subtitle "World leading innovations in fluoropolymer processing" and the website "www.fluorocarbon.co.uk". The grid includes video feeds for participants: Nicusor Voinea, Cristian Nasta, Barni, Preda Mircea, NICOLETA MADRINSICU, MATTEO SALIANI, and "You". A "20 others" icon is also present. The meeting title at the bottom is "1:02 PM | MID-DAY: Matematica întâlnește Industria".

1.8: Αξιολόγηση των εκδηλώσεων MID-Day: Σχόλια μαθητών

Μετά την ολοκλήρωση των εκδηλώσεων MID-Day στις πέντε χώρες που αναφέρθηκαν παραπάνω, οι συμμετέχοντες στην εκδήλωση έλαβαν μια αξιολόγηση για να έχουν πρόσβαση στην ικανοποίησή τους για τις δραστηριότητες MID-Day.

Η συνολική ανατροφοδότηση από τους μαθητές ήταν πολύ θετική και ενθαρρυντική. Οι μαθητές εξέφρασαν την ικανοποίησή τους για την ευκαιρία να εργαστούν ως μέλη της ομάδας και να τους ανατεθούν ρόλοι εντός των ομάδων τους. Πίστευαν ότι ήταν ευχάριστο και εποικοδομητικό. Όλοι οι μαθητές από όλες τις εκδηλώσεις είπαν ότι επωφελήθηκαν από τη δραστηριότητα MID-Day για διάφορους λόγους. Ένας από τους λόγους ήταν ότι εργάστηκαν σε ένα πραγματικό πρόβλημα ως μέλος μιας ομάδας που ανέπτυξαν τις δεξιότητες επικοινωνίας και παρουσίασης, ότι συνειδητοποίησαν πόσο περίπλοκα είναι τα πραγματικά προβλήματα και πώς τα μαθηματικά μπορούν να παρέχουν μια λύση. Οι μαθητές ανέφεραν επίσης ότι η ευκαιρία να παρουσιάσουν το έργο τους κατά τη διάρκεια της μορφής MID-Day αποδείχθηκε σημαντική για αυτούς - χάρη στην οποία θα μπορούσαν να βελτιώσουν και να αναπτύξουν τις κοινωνικές δεξιότητές τους όπως: ανταλλαγή πληροφοριών, επιχειρηματολογία. Τέλος, όλοι οι μαθητές ανέφεραν ότι ήταν μια μοναδική εμπειρία για αυτούς να γνωρίσουν τον κόσμο της βιομηχανίας!

Αυτό που είναι σημαντικό να αναφέρουμε είναι ότι όλοι οι μαθητές από όλες τις χώρες συμφώνησαν ότι τα προβλήματα της MID-day ήταν ένας πολύ πιο ενδιαφέρον και διαδραστικός τρόπος μάθησης από τα μαθηματικά προβλήματα που αντιμετωπίζουν στα σχολεία τους. Επιπλέον, η πλειοψηφία των μαθητών ανέφερε ότι κατάφεραν να μάθουν κάτι νέο από την εκδήλωση mid-day.

Όταν οι μαθητές ρωτήθηκαν για τη γνώμη τους σχετικά με τα πλεονεκτήματα, την πρακτικότητα και τις δυσκολίες υιοθέτησης της στα σχολικά προγράμματα σπουδών, κάποιιοι είπαν ότι είναι απαραίτητο να υπάρχουν αυτού του είδους τα προβλήματα στα σχολικά προγράμματα σπουδών και ότι οι μαθητές θα αποκτήσουν μεγαλύτερη αυτοπεποίθηση σχετικά με τα Μαθηματικά εργαζόμενοι σε ομάδες και θα αποκτήσουν πραγματική εργασιακή εμπειρία. Πιστεύουν ότι δεν θα ήταν δύσκολο να υιοθετήσουν αυτή την κουλτούρα στο σχολείο. Από την άλλη, μερικοί μαθητές εξέφρασαν την άποψη ότι αν και για αυτούς αυτό ήταν μια σπουδαία εμπειρία, το επίπεδο των μαθηματικών στο σχολείο είναι χαμηλό και πως στα σχολεία το ενδιαφέρον για τα μαθηματικά είναι χαμηλό και υψηλότεροι στόχοι θα μπορούσαν να αποξενώσουν περαιτέρω μερικούς μαθητές από τα Μαθηματικά.

Όσο για ορισμένες ιδέες για τη βελτίωση της MID-Day, οι μαθητές θα ήθελαν να δουν περισσότερες επιλογές προβλημάτων στο μέλλον με ποικίλες δυσκολίες. Διαπίστωσαν ότι τα προβλήματα και η όλη διαδικασία ήταν πολύ δύσκολα και εξέφρασαν την επιθυμία να συμπεριληφθούν τέτοια είδη εκδηλώσεων και δραστηριοτήτων στα σχολικά προγράμματα σπουδών, όπου αυτό είναι δυνατόν.

Part C: MiD Day - MATEMATYKA SPOTYKA PRZEMYSŁ – Projekt planu aktywności (PL)

Jeśli zidentyfikowałeś uzdolnionych uczniów, przekonałeś przedstawicieli biznesu i przemysłu, że ich zdolności myślenia mogą okazać się pomocne, zmotywowałeś naukowców, aby Cię wsparli, teraz musisz zaplanować i zaprojektować MiD Day „Matematyka spotyka Przemysł”. Oto wskazówki dotyczące planowania tego wydarzenia.

Po pierwsze, rozmawiając z partnerem przemysłowym, musisz zdecydować, jaki rodzaj wydarzenia chcesz przeprowadzić. Oto kilka propozycji. Dla każdego z nich będziesz musiał zaplanować kurs, aby wyrównać poziom uczniów w odpowiednim stopniu, a my przedstawimy przybliżony plan lekcji, aby to zrobić.

- **Badanie statystyczne** zainteresuje Twojego partnera przemysłowego, który chce wiedzieć więcej o potrzebach swoich klientów lub pracowników, badanie rynku lub zasobów ludzkich jest stosunkowo łatwe do skonfigurowania, zabawne i kreatywne w projektowaniu i może prowadzić do nieoczekiwanych wyników, z niezwykłego punktu widzenia: „jak przekształcić otwartą przestrzeń w kontekście obostrzeń COVID 19?”. Plan lekcji w zakresie statystyki i metodologii ankiet jest konieczny przed faktycznym początkiem pracy.
- **Ścieżka naukowa** na terenie przemysłowym to świetny pomysł na promocję swojej pracy przez branżę w czasie wizyt uczniów, a także sposób, aby uczniowie zaczęli tworzyć i rozwiązywać różnorodne łamigłówki i przyjrzeć się swojemu otoczeniu naukowym okiem: „Jak duży jest ten zbiornik? Ile czasu zajmie jego napełnienie?” Plan lekcji powinien koncentrować się na metodologii modelowania i pomiaru, odkrywając na przykład, że własne ciało jest dość precyzyjnym narzędziem do szacunków ilościowych.
- Porozmawiaj z partnerem przemysłowym. Na przykład klaster mobilności w Lyonie, współpracujący z Izba Matematyki i Informatyki, interesuje się **Sztuczną Inteligencją** i pojazdami autonomicznymi. Scenariusz lekcji będzie obejmował podstawy robotyki i wprowadzenie do statystyki w ujęciu bayesowskim i naukę wzmacniania, zwłaszcza z analogową maszyną, która uczy się, jak cię pokonać w grze Nima!
- Jeśli nie masz bezpośredniego kontaktu z partnerem przemysłowym, nie zapomnij o instytucjach kształcących przyszłych nauczycieli i poproś studentów, aby przygotowali dla Ciebie gry pedagogiczne: pozwól im wymyślić nowe sposoby uczenia poprzez grywalizację niektórych treści. Ucz się od nich! Plan lekcji powinien uczyć neuronalnych podstaw poznania, grywalizacji, wyznaczania celów i podzadań, definiowania wyzwań i poziomów, sposobów współpracy, pomagania uczniom w lepszym zrozumieniu ich własnego procesu poznawczego. Nigdy nie rozumiesz pojęcia lepiej niż dzięki konieczności wyjaśniania go, zwłaszcza komputerowi!

Po ustaleniu głównego tematu musisz zaplanować ścieżkę lekcji, aby poprowadzić uczniów do miejsca, w którym zdobędą wymagane kompetencje w zakresie wykonywanych zadań. Następnie musisz przeprowadzić lekcję wprowadzającą do projektu. Ten pierwszy etap powinien być spokojny i radosny, studenci pracując w zespołach powinni dobrze poznać się nawzajem.

Podczas głównego etapu MID -Day studenci odwiedzają teren przemysłowy lub dyskutują z przedstawicielami branży i wspólnie pracują nad swoim projektem. Dzięki temu wrócą do domu z mnóstwem obserwacji, pomysłów i planów.

W ostatniej fazie praca ta jest prezentowana partnerowi przemysłowemu. Wyniki pracy mogą być również prezentowane w trakcie innych wydarzeń takich jak szkolne targi edukacyjne, spotkanie partnerskich szkół, tygodnie integracyjne, etc.

Partie C: Guide pour la tenue d'un jour Math-Industrie

Mathematics Industry Day (MID-Day) (FR)

Chapitre 1: Guide d'un MID-day

Vous devez d'abord identifier les étudiants à haut potentiel, puis convaincre un partenaire industriel que leurs capacités de réflexion latérale pourraient être mises à profit. En même temps, vous devez constituer une équipe d'universitaires et d'enseignants motivés pour vous soutenir, puis vous devez planifier et concevoir ce MID-day (une journée où les mathématiques rencontrent l'industrie). Ce chapitre fournira les lignes directrices pour la planification et la conduite d'un tel événement.

1.1: Définition and Glossaire

Partenaire : Dans ce document, lorsque nous parlons de partenaires, nous entendons une industrie prête à investir une journée ou deux à discuter avec des universitaires, des enseignants et des étudiants afin de définir et de résoudre un problème industriel.

Problème industriel : Un problème approprié présenté par le partenaire industriel, qui est compréhensible par les étudiants, les enseignants et les universitaires. Il doit être adapté des deux côtés, aborder certaines questions utiles du côté de l'industrie et être stimulant, mais faisable du côté des étudiants. L'état du problème est le résultat d'un brainstorming commun entre le partenaire industriel, les universitaires et les enseignants, en amont du MID-Day. Une liste des problèmes possibles est fournie dans la section 1.4 ci-dessous.

MID-Day : La tenue d'une Journée Mathématiques et Industrie, où les élèves abordent un problème mis en place par un partenaire industriel et adapté par les universitaires et les enseignants. Il pourrait s'agir davantage d'une idéation, du raffinement d'une idée, que de la solution complète d'un problème.

1.2: Méthodologie, exigences et lignes directrices pour les enseignants (l'enseignant en tant que mentor et co-créateur)

Votre responsabilité principale, en tant qu'enseignants, est de sélectionner et de préparer les élèves de manière à les rendre aptes à un MID-Day fructueux. Pendant le MID-Day, vous devez faciliter le travail des élèves. Cela signifie que les élèves sont d'abord sélectionné-e-s, que ce soit à l'aide d'un test écrit ou sur la base d'une présélection par une association des parents d'élèves, les notes à l'école, la sélection des olympiades ou des clubs de mathématiques... Il faut obtenir des accords formels des parents et des écoles pour la participation de chacun.

Après votre première discussion avec le partenaire industriel et les universitaires, vous devez décider du type de MID-day que vous souhaitez réaliser. Vous trouverez ci-dessous quelques propositions. Vous devrez mettre en place un cours de mise à niveau, basé sur un plan de cours que nous fournissons, pour élever les étudiant·e·s au niveau adéquat. Cela implique également d'ajuster les rôles en fonction des niveaux : certains étudiant·e·s plus avancé·e·s ou plus matures peuvent agir comme tuteurs pour les plus jeunes, sachant s'effacer si besoin, comme guide sur le côté plutôt que comme sage sur la scène.

L'animation pendant le MID-Day consiste à soutenir les étudiants intimidés par l'ampleur de la tâche, à renforcer la confiance en soi, à rassurer sur le fait que leurs connaissances ne sont pas insuffisantes pour commencer à réfléchir au problème, et à faciliter la communication, en s'assurant que personne ne se sent en insécurité ou n'impose son point de vue, que chaque voix est entendue, que chaque élève se sent nécessaire et valorisé·e d'une manière ou d'une autre. La division du problème en sous-tâches doit être planifiée à l'avance à cette fin. Enfin, vous devez rendre compte de comment les choses se sont passées, en observant la régulation et l'autorégulation, comment chaque élève a réagi, ce qui déclenche ou détériore sa motivation, son sentiment d'appartenance, ce qui a été proposé et créé.

Après le MID-Day, vous devez accorder une attention particulière à la manière dont le matériel soulevé pendant le MID-Day peut être utilisé dans le cours de mathématiques de tous les jours, en le reliant aux connaissances institutionnelles. Recherchez également des occasions pour les élèves de partager leur travail avec d'autres lors de foires, de concours, de forums, de clubs ou en organisant une exposition dans votre école. Cela aidera à recruter des étudiants pour l'année à venir. N'oubliez pas de mentionner que la participation à un MID-Day peut être valorisée par les étudiants dans leur CV lors d'une demande de bourse. Cet événement va faire naître et nourrir leur réseau professionnel !

1.3: Méthodologie, exigences et lignes directrices pour les universités soutenant les journées MID

Les partenaires universitaires doivent constituer l'équipe d'étudiants qui travaillera sur le problème industriel. Cela signifie travailler avec les étudiants mais aussi former les étudiants à la gestion du travail en équipe. En collaboration avec les partenaires industriels et les enseignants, ils doivent façonner le problème de manière à ce qu'il puisse être travaillé par les étudiants.

Votre principale responsabilité est de mettre en place, d'organiser et de gérer le MID-Day et les réunions préparatoires en amont. Une interaction typique avec les enseignants et les élèves comportera trois phases :

Lors du 1er rendez-vous : Vous allez rencontrer de manière informelle les élèves et leurs professeurs, les motiver, constituer une équipe autour d'un matériel mathématique original, créatif et ludique. Vous pouvez contacter et recruter les participants d'un club de mathématiques, à une expo-sciences, à une journée des mathématiques, aux concours de mathématiques, aux réunions inter-écoles, etc.

Lors d'une 2e réunion, vous améliorerez le niveau des étudiants sélectionnés pour constituer le groupe en équipe. Cela signifie que vous leur apprendrez les outils pour travailler en équipe (voir ci-dessous).

La 3ème rencontre consistera en le MID-Day proprement dit. Vous devrez sélectionner des locaux suffisamment grands et pratiques pour la réunion, que ce soit à l'université ou sur le site du partenaire industriel, avoir des pauses café et déjeuner à portée de main, etc. Une version hybride, avec des téléconférences en ligne, pourrait être de mise si aucune réunion physique n'est envisageable.

Vous devrez faciliter la communication et le travail d'équipe en posant les bonnes questions pour que chacun se sente à la hauteur et productif. Attention à ne pas chercher les réponses par vous-même, il faut vous fier aux élèves! Assurez-vous que les élèves sont pertinents et au centre de la mi-journée. Vous n'êtes pas ici en tant qu'enseignant qui enseigne aux étudiants, ils doivent être considérés comme des collègues chercheurs stagiaires.⁹

Outils de base pour travailler en équipe:

- Explicitiez les divergences. Tout en gardant un environnement sécurisé, parvenez à un débat fructueux : ne votez pas, ne faites pas de compromis, ne sélectionnez pas au hasard, évitez la moyenne et le marchandage mais visez l'unanimité.
- Écoutez et impliquez : Ne laissez personne imposer son point de vue mais argumentez, ne rejetez pas mais proposez des réfutations.
- Ne personnalisez pas les débats, ne prenez pas les choses personnellement. Cherchez des solutions.
- N'acceptez les solutions que si elles sont comprises et partagées par tous.
- Donner une formation légère sur ces thèmes, tant aux étudiants qu'aux enseignants: cela augmente la probabilité d'obtenir des solutions originales et une équipe productive.

⁹ Hall, Jay (1971) Decisions, Decisions, Decisions, Psychology Today, November, 51-54, 84-88.

1.4: Exemples de MID days

- Discutez avec votre partenaire industriel. Les **problèmes d'optimisation** peuvent être trouvés dans de nombreuses entreprises : que le partenaire industriel coupe du verre, contrôle numériquement des robots, gère un entrepôt ou organise une chaîne d'approvisionnement, il a sûrement un problème d'optimisation ouvert de base qui doit être clarifié s'il n'est pas résolu.
- Une **enquête statistique** intéressera votre partenaire industriel, il voudra peut-être mieux connaître les besoins de ses clients ou de ses employés, et un projet d'étude de marché ou de ressources humaines est relativement facile à mettre en place. Amusant et créatif à concevoir, il peut conduire à des résultats inattendus: « comment transformer cet open-space morne dans le contexte covid ? ». Un plan de leçon sur les statistiques et la méthodologie d'enquête s'impose avant de se mettre au travail.
- Un **Rallye Scientifique** sur le site industriel est un excellent moyen pour les industriels de valoriser leur travail lors de visites de classes, et pour les élèves de devenir des poseurs de problèmes et de regarder leur environnement avec un œil scientifique : « quelle est la taille de ce réservoir ? combien de temps faudra-t-il pour le remplir ? » Le plan de cours doit se concentrer sur la modélisation et la méthodologie de mesure, en découvrant par exemple que votre propre corps est un outil assez précis pour les estimations de quantités quotidiennes.
- Si vous n'avez pas de contact direct avec un partenaire industriel, n'oubliez pas l'industrie de **l'éducation** et faites mettre en place par les étudiants des jeux pédagogiques pour vous : laissez-les inventer de nouvelles façons d'enseigner en ludifiant certains de vos contenus. Apprenez d'eux ! Le plan de cours doit enseigner les bases neurales de la cognition, la gamification, la détermination des objectifs et des sous-objectifs, la définition des défis et des niveaux, les façons de collaborer, en aidant les élèves à mieux comprendre leur propre processus cognitif. Et on ne comprend jamais mieux un concept que quand on doit l'expliquer, surtout à un ordinateur !

Une fois le thème principal défini, vous pouvez planifier le parcours de la leçon pour amener les élèves d'où ils en sont aux compétences requises concernant les tâches à accomplir. C'est-à-dire: vous devez mener la leçon dans une introduction au projet. Cette première journée à l'école doit être détendue et joyeuse, les élèves travaillant en équipe et apprenant à se connaître.

Une fois le problème affiné avec le partenaire, vous devez l'envoyer aux étudiants à l'avance avec une possibilité de réagir, d'interagir, avec des questions de clarification.

Lors de la grande journée du MID-Day, les étudiants visitent le site industriel ou du moins échangent avec des représentants de l'industrie et travaillent ensemble sur leur projet durant la matinée.

Voici un horaire possible pour le MID-Day :

- 9h-9h30 : Bienvenue, brise glace
- 9h30-10h30 : Constitution d'équipe avec une activité de groupe comme « perdu sur la lune » ou similaire. <https://netmind.net/play-the-moon-landing-exercise/>
- 10h30-10h45 Pause
- 10h45-11h45 Discussion en sous-groupes, autour de la problématique du MID-Day, compréhension de la problématique, premières pistes de réflexion...
- 11h45-12h15 Première synthèse sur les stratégies possibles face aux enjeux, en fixant les équipes sur des tâches précises.
- 12h15-13h45 Pause déjeuner
- 13h45-15h45 Travail en sous-groupes sur des stratégies et des tâches spécifiques.
- 15:45-16 Pause
- 16-16:30 Préparation de la synthèse et sa présentation [Les étudiants]
- 16h30-17h15 Présentation devant les partenaires industriels, débat

1.5: Évaluation des résultats, choses à faire ou à éviter

Pendant le MID-Day:

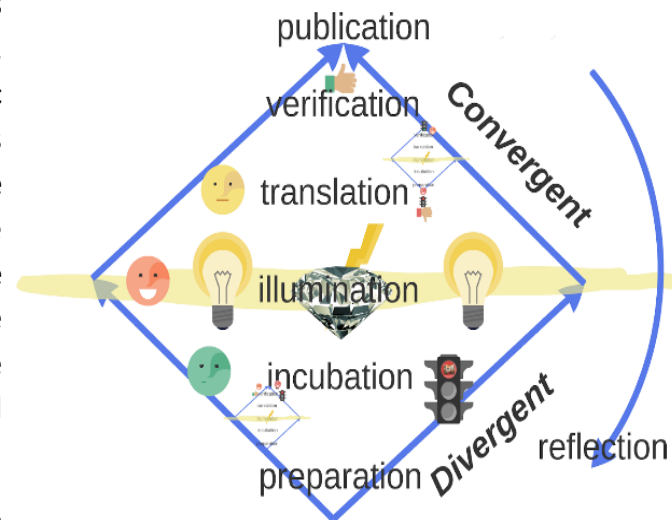
Attention à ce que chaque étudiant·e se sente appartenir à un petit groupe, maximum 8 étudiant·e·s, qui doit être formé dans les activités préparatoires, par exemple avec des étudiant·e·s plus âgé·e·s supervisant le groupe en tant que coach, faisant en sorte que chacun se sente nécessaire et inclus. Il faut accorder une attention particulière aux élèves peu sûr·e·s en posant des questions appropriées, en soulignant et en développant les capacités particulières de chaque élève, en augmentant la confiance en soi. Le problème industriel est ainsi analysé sous tous les angles, de sorte qu'il est possible de répartir les tâches partielles entre tous les membres de l'équipe, favorisant la collaboration de tous les membres de l'équipe.

L'évaluation doit se concentrer sur ces aspects : que chaque élève se sente pris en compte en tant que penseur et créateur individuel. Bien sûr, toutes les idées ne peuvent pas aboutir à un produit réussi, peut-être aucune production ne sortira du MID-Day, et cela ne doit pas être ressenti comme un échec, mais les potentialités et la créativité des idées et des productions doivent être mises en avant. Si vous avez besoin de chiffres, ou souhaitez attribuer des trophées individuels, vous pouvez objectiver l'évaluation de la créativité du processus d'idéation qui s'est

déroulé selon les quatre dimensions décrites par Guilford : Fluidité, Flexibilité, Élaboration et Originalité :

- La fluidité est la capacité de produire beaucoup, d'élaborer beaucoup d'exemples.
- La flexibilité est la capacité à changer de stratégie quand on est bloqué, sérendipité et adaptation sont clairement les maîtres mots ici.
- L'élaboration approfondit une question, cherche les détails fins, est exhaustive dans sa recherche.
- L'originalité est le type de créativité que vous reconnaissez quand vous la voyez : vous n'avez jamais rien vu de tel, c'est nouveau et provocateur.

Puisqu'on parle d'idéation, sachez que nous ne sommes pas dans la tête des élèves; pour être pleinement créative, une idée doit passer par tout le cycle de la créativité avec les deux moments de phases divergentes et convergentes : de la préparation à l'incubation, puis illumination (le moment *Eureka*), mais cette idée doit sortir de la tête de l'élève et être traduite en mots, en croquis, etc., pour être vérifiée, puis publiée, proposée aux collègues pour voir ce qu'ils en pensent, s'ils le valident. Et en fin de compte, cette idée peut être réfléchiée et intégrée dans un plus grand cycle d'idées afin de résoudre le problème en question.



Il ne faut pas confondre la créativité avec seulement une pensée divergente : une phase de brainstorming est acceptable, mais à un moment donné, elle doit s'éloigner de la rêverie et converger vers une solution réellement réalisable, et il y a un besoin d'autant de créativité dans cette phase convergente. De même, le moment d'illumination ne doit pas être surévalué, la poésie et l'industrie ne peuvent se mélanger que jusqu'à un certain point, ce qui semble être une idée originale doit se traduire par des schémas réels. Ces points subtils doivent être pris en considération par l'équipe afin de favoriser un état d'esprit productif chez les étudiants.

Après le MID-Day:

Tous les résultats ne se verront pas immédiatement, particulièrement pas pendant le MID-Day. Les élèves reviendront à la maison avec beaucoup d'observations, d'idées et de plans. Dans la dernière phase, ce travail peut être retravaillé et présenté à nouveau au partenaire industriel. Il est important d'organiser un moyen d'échanger avec les autres. Celui-ci peut être utilisé par la suite, pour des salons scolaires, des forums, des jumelages d'écoles, des semaines d'intégration, des concours, etc. afin de montrer ce qui a été fait.

1.6: Pourquoi l'industrie devrait s'impliquer, avantages pour l'industrie

Ce point de l'intérêt du partenaire industriel doit être éclairci au préalable mais il est bon de le revoir à la fin du MID-Day et de parcourir la check-list : Toutes les parties ont-elles bénéficié du MID-Day ? La pensée latérale a-t-elle vraiment débloqué une situation de créativité ou simplement l'industrie a-t-elle pu montrer son entreprise à des étrangers.

Les bénéfices potentiels sont clairement différents selon le type de MID-Day qui a été réalisé. Sur les exemples génériques, nous avons par exemple identifié :

- L'enquête de recherche statistique a-t-elle pu mettre en évidence des éléments qui n'avaient pas été vus auparavant, ou s'agissait-il simplement d'un moyen de communiquer, que ce soit en interne ou avec des clients potentiels ? Permettre aux clients ou aux collaborateurs de s'exprimer, de voir qu'ils sont pris en compte est déjà quelque chose qui peut être valorisé.
- Le Rallye Scientifique est-il ludique et informatif ? Peut-il être utilisé avec des clients potentiels, avec une classe ? Ouvre-t-il les yeux des participants sur les défis que l'industrie relève ? Dans le cas d'une visite touristique d'un site, le partenaire découvre-t-il le site d'un œil nouveau ? Atteindra-t-il son objectif de tourisme scientifique ?
- En ce qui concerne l'industrie de l'éducation, les étudiants ont-ils mis en place des jeux pédagogiques intéressants, informatifs et amusants ? Ont-ils appris quelque chose en concevant le jeu pédagogique ?

Il faut donner l'occasion au partenaire industriel d'entrer en contact avec des élèves à haut potentiel et éventuellement de travailler avec eux à l'avenir.

1.7: Implementation du MID – Day dans les pays partenaires:

Des MID-Days ont été mis en œuvre dans cinq pays européens: Chypre, la France, l'Allemagne, la Pologne et la Roumanie. L'idée derrière un MID-Day est que l'application des mathématiques dans la vie réelle est mis en avant dans tous les programmes scolaires de nos jours et encore plus pour les élèves doués. Ils/elles peuvent contribuer à l'innovation qui pourrait améliorer l'humanité et en même temps promouvoir l'entrepreneuriat. Cette rencontre avec l'industrie est conçue pour proposer aux élèves des problèmes non résolus de l'industrie dans le but de travailler en équipe ou individuellement et d'utiliser les mathématiques afin de proposer un ensemble de solutions possibles aux problèmes donnés. De plus, les étudiants ont eu l'occasion de collaborer entre eux et de travailler ensemble vers un objectif commun, de rencontrer des chercheurs de l'enseignement supérieur ainsi que des industriels et des enseignants du secondaire. Ils peuvent surtout avoir des idées et s'inspirer de la façon dont ces problèmes pourraient être utilisés dans le cadre de leur enseignement régulier.

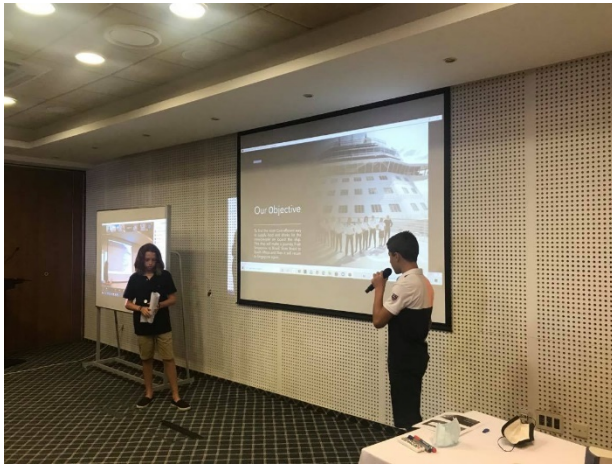
La préparation et la mise en œuvre de l'événement ont varié dans chaque pays. Divers problèmes industriels ont été présentés, que les étudiants ont été mis au défi de résoudre. Par conséquent, ci-dessous, nous analyserons la préparation et la mise en œuvre de chaque événement MID-day par pays. De cette façon, vous pourrez vous faire une idée sur la méthodologie MID-day en partant d'exemples et de suggestions.

Chypre

Préparation: La préparation de cet événement a commencé deux mois à l'avance lorsqu'une lettre d'invitation/motivation a été envoyée aux étudiants qui aiment les mathématiques et qui étaient potentiellement intéressés par une telle activité. Entre-temps, l'équipe pédagogique INNOMATH de l'Ecole Privée Héritage a contacté des entreprises locales, suite à l'organisation d'un Focus Group, et leur a proposé de se joindre à cet événement en proposant aux étudiants une problématique concrète de l'industrie. Deux entreprises ont fourni chacune un problème dans le cadre du MID-Day: Le problème A a été fourni par V-ships, l'une des plus grandes compagnies maritimes au monde. Le problème décrit un cas spécifique d'un navire-citerne faisant du commerce entre Singapour et le Brésil qui doit être approvisionné afin d'avoir des provisions à bord à tout moment. Dans ce projet, les étudiants devaient agir au profit du prestataire de restauration pour tenter de formuler le plan de réapprovisionnement le plus rentable en s'assurant que les tarifs convenus soient respectés et que le navire disposera à tout moment d'un stock suffisant à bord pendant le voyage; et sous ces contraintes, la question est de maximiser le profit de l'entreprise de restauration. Le problème B a été donné par une entreprise d'architecture et les étudiants ont été invités à prendre la place d'un architecte, à comprendre le processus et à trouver la meilleure solution possible pour servir un client en fonction des besoins de sa maison, des matériaux à utiliser et du budget disponible.

Travail des étudiants : De nombreux élèves ont choisi de participer au MID-Day. C'était l'occasion pour eux de penser de manière créative et hors des sentiers battus, soit individuellement, soit en équipe, afin de trouver des solutions possibles aux problèmes posés dans une activité en dehors de la salle de classe. En raison des restrictions de Covid19 et des contraintes logistiques, deux réunions en ligne ont été organisées par les enseignants de l'école Héritage, les élèves participants, les représentants de l'entreprise et la Société mathématique. Les réunions en ligne ont eu lieu au début et à la mi-juillet. L'objectif des rencontres était que les élèves constituent leurs équipes, se familiarisent avec les problématiques, obtiennent des compléments d'explications et d'éclaircissements de la part des enseignants ainsi que des représentants de l'entreprise. Des directives générales pour le bon déroulement du MID-Day et les exigences pour les présentations des élèves ont également été données. Après la première réunion, les élèves ont eu quelques jours pour décider de leur problème préféré et ont commencé à travailler dessus. Le problème A a été de loin le plus populaire, il a été choisi par tous les élèves. Ensuite, une autre réunion en ligne a eu lieu pour les élèves et les enseignants afin de discuter de l'avancement de la conception de la solution. Les élèves ont été conseillés sur la manière d'organiser leur travail et de se répartir la charge de travail au sein de l'équipe. Les élèves ont également eu la possibilité de demander plus d'assistance et des clarifications qui pourraient les aider à travailler sur une proposition de solution.

Le MID-DAY proprement dit: Le mercredi 28 juillet 2021, l'événement s'est déroulé à Agros en présence d'élèves, de parents, d'enseignants, de chercheurs de l'enseignement supérieur et de partenaires du projet. Les élèves ont présenté leurs solutions et à la fin de chaque présentation le représentant de V-ships, M. Loucas Chrysochos, donnait son avis sur les solutions proposées par les élèves. Les autres participants étaient libres de poser des questions aux élèves. Toutes les solutions présentées étaient différentes et cela a initié une discussion fructueuse entre les élèves participants et le public. À la fin, le représentant de l'industrie a exprimé son enthousiasme pour le travail acharné des équipes, leurs excellentes compétences en présentation et la manière analytique de présenter leur solution. Il a mentionné que c'était une expérience formidable pour lui de participer à cet événement et de rencontrer de tels élèves qui ont fait un excellent travail. De plus, il a exprimé son intérêt à travailler un jour avec eux.



France

Les élèves participant au MID-Day portent deux casquettes, ils sont conscients qu'ils ne font pas que participer mais qu'ils testent également le plan pédagogique innovant. Les élèves ont été amenés à adopter un état d'esprit créatif, où les mathématiques sont considérées comme un outil concret pour résoudre des problèmes de la vie réelle d'une valeur économique.

Les élèves se sont mis en équipe grâce à des brise-glace, puis ont installé l'application MathCityMap et exécuté un parcours mathématique local.

Ils ont passé la majeure partie de la journée sur le terrain, dans le quartier Renaissance du patrimoine UNESCO, écoutant les histoires que le guide professionnel racontait, prenant des notes, mettant en place des questions en équipe, essayant des idées, laissant d'autres membres de l'équipe vérifier la faisabilité et l'intérêt des tâches... De retour à l'université, les tâches étaient encodées dans le système MathCityMap.

L'enjeu principal était de répondre aux besoins du représentant de l'industrie touristique : les étudiants ont discuté des meilleures façons de s'adresser à différentes cibles, familles, classes, groupes d'amis. Comprendre les contraintes du guide professionnel, insuffler suffisamment d'informations ludiques et révélatrices dans les questions scientifiques sont les objectifs

principaux du parcours mathématique qui a été produit.

La deuxième partie du MID-Day était le retour d'expérience au partenaire industriel. Les étudiants ont eu l'occasion de se rassembler afin de mettre en place leur exposé. La journée s'est déroulée dans le cadre du projet transnational InnoMath, une réunion qui a eu lieu à Lyon mi-décembre 2021 sur le campus de l'UCBL et où les partenaires ont pu tester le rallye. Le lendemain du parcours, les étudiants ont justifié les choix qu'ils ont faits devant le jury composé du représentant de l'industrie et des partenaires internationaux du projet InnoMath.



Allemagne

Préparation: La préparation de l'événement de la mi-journée a commencé 6 mois à l'avance par une réunion avec des partenaires de l'industrie pour discuter des tâches potentielles. La réunion du groupe de discussion, composé d'enseignants, de scientifiques et de partenaires de l'industrie, s'est réunie et a évalué les idées et l'horaire proposé pour le MID-Day. Plus tard, les écoles axées sur les mathématiques et les sciences naturelles ont été invitées à participer à l'événement. En collaboration avec la Stiftung Planetarium Berlin, il a été décidé de poser aux étudiants le problème de savoir comment les changements de saisons - le changement climatique - peuvent être rendus statistiquement visibles et comment des prévisions peuvent être faites. En particulier, les modèles utilisés devaient être évalués et réfléchis.

Le travail des élèves: 14 classes (420 élèves) d'écoles à dominante mathématiques et sciences naturelles ont participé au MID-Day. En raison des restrictions de Covid19, les élèves ont dû se réunir dans leurs écoles et n'ont pas pu rencontrer les élèves d'autres écoles pour la préparation de la présentation. Les préparatifs ont eu lieu en septembre et les élèves ont reçu du matériel pour se familiariser avec les problèmes, qui a été préparé par les chercheurs et les partenaires industriels. Les professeurs soutenaient les élèves et les encadraient. Des directives générales pour le bon déroulement du MID-DAY et les exigences pour les présentations des élèves ont également été données. Après la première réunion, les élèves ont eu quelques jours pour décider de l'orientation de leur solution et de leur présentation, qui a ensuite été discutée en classe. Les

élèves ont décidé comment organiser leur travail et comment répartir la charge de travail au sein des équipes. Les élèves ont également eu la possibilité de demander plus d'assistance et des clarifications qui pourraient les aider à travailler sur leur proposition.



Pologne

Préparation: Les préparatifs ont commencé un mois à l'avance. Les élèves devaient se familiariser avec la tâche présentée par l'office de tourisme. Elle consistait à construire un algorithme permettant de créer un plan de voyage pour les étudiants visitant Cracovie. Ce plan devait tenir compte d'un certain nombre de conditions, telles que le budget, la durée, l'âge des participants, le choix d'attractions optionnelles et autres.

Travail des élèves: L'événement réunissant des groupes d'étudiants des mêmes classes, le travail s'est déroulé en présence. Leur travail consistait à collecter des données pertinentes, telles que le prix des billets, la distance entre les excursions, les tarifs de location d'autocars, le prix des transports en commun et autres. Dans la deuxième phase des préparatifs, la tâche était de créer un programme pour simuler les coûts de l'événement. Des méthodes traditionnelles et des technologies de l'information modernes ont été utilisées pour la préparation. Certains travaux ont également été effectués à distance. Dans la phase finale de préparation, les membres du groupe se sont concentrés sur la présentation de leurs réalisations, à savoir la présentation et la formation de leurs discours.

Le MID-Day proprement dit : L'événement a été divisé en trois parties : Dans la première partie, le projet INNOMATH a été présenté, ses objectifs, hypothèses et les effets développés. La deuxième partie consistait en la présentation des travaux des élèves. Tous les groupes ont présenté les résultats de leurs préparations. Après chacune d'elles, experts et invités ont eu l'occasion de s'exprimer sur les propositions. La troisième partie était une discussion sur des sujets liés au projet et son impact sur l'approche moderne de l'enseignement.



Roumanie:

Remarque : Cet exemple de MID-Day est écrit depuis le point de vue des élèves

Tout au long du projet INNOMATH, nous avons découvert de nombreuses choses nouvelles qui nous ont montré à quoi ressemble la vie après le lycée et comment les mathématiques peuvent être utilisées dans des situations réelles. Afin de mieux comprendre à quel point les mathématiques appliquées sont essentielles au bon fonctionnement des entreprises et des installations industrielles, nous nous sommes lancés dans un voyage pour tenter de résoudre un problème auquel une entreprise locale était confrontée, en utilisant nos connaissances.

Ainsi, nous avons visité Fluorocarbon Polymers et discuté avec son directeur. Il nous a expliqué qu'ils produisent et expédient diverses pièces et matériaux qui servent à toutes sortes de fins dans l'industrie. Nous avons ensuite suivi la chaîne de production du début à la fin, dans l'espoir de repérer les façons dont l'entreprise perd de l'argent et/ou des ressources afin que nous puissions trouver une solution. Il s'est avéré assez difficile de trouver des problèmes dans leur processus, mais au final, nous avons décidé d'essayer d'optimiser le transport des matières premières qu'ils utilisent et des produits finis vers leurs acheteurs.

Les problèmes d'optimisation des transports sont difficiles à résoudre et nécessitent des compétences de niveau universitaire. Par conséquent, nous avons fini par apprendre des éléments d'algèbre avancée qui nous a seulement rapprochés de la capacité à résoudre de tels problèmes, tout en nous aidant à résoudre des problèmes d'algèbre linéaire de niveau secondaire plus facilement.

Dans la classe, notre professeur nous a divisés en deux groupes : un qui apprendrait et utiliserait les méthodes habituellement enseignées au lycée pour calculer des matrices inverses, déterminer le rang d'une matrice et résoudre des systèmes d'équations linéaires ; et un qui apprendrait les méthodes avancées. L'objectif était de comparer les différentes méthodes à la fin du cours et de déterminer celles qui sont plus rapides et plus faciles à utiliser.

Au fur et à mesure que nous pratiquions des exercices de résolution, nous étions encouragés à nous entraider à comprendre et à apprendre plus rapidement. En effet, il était parfois plus facile de poser des questions à mes camarades de classe sur les nouveaux concepts plutôt que de demander au professeur, et leur perspective d'élève leur permettait d'avoir un certain aperçu utile du problème. L'aspect le plus agréable de cette méthode d'apprentissage était le fait qu'elle ressemblait moins à un apprentissage qu'à un jeu, car nous échangeons des idées les uns avec les autres.

Au final, les deux équipes ont présenté et échangé les méthodes qu'elles avaient utilisées pour se faire une idée de l'autre moitié de cette histoire et pouvoir tirer des conclusions plus facilement. Nous avons tous convenu que les méthodes avancées n'étaient pas aussi difficiles à comprendre que nous le pensions auparavant, mais la manière dont vous les apprenez est vraiment importante. En repensant à tout ce que nous avons vu et accompli, je me sens confiant d'avoir choisi d'étudier l'ingénierie à l'université et d'être mieux préparé à affronter le monde réel.

Le MID-Day:

The image shows a Zoom meeting interface. The main window displays a presentation slide titled "Ce este MID-Day?". The slide includes the "m:omath" logo, the text "Co-funded by the Erasmus+ Programme of the European Union" with the EU flag, and a definition: "Ce este MID-DAY? MID-Day: o zi reală a matematicii și a industriei, când elevii abordează o problemă stabilită de un partener industrial și analizată / adaptată de academicienii / cercetători și facilitată de profesori." Below the text is a photo of students in a classroom. The slide footer shows the project number: "Project Number: 2019-1-DE03-KA201-059604".

On the right side of the Zoom window, there is a video call grid. The top video shows a woman named "MARINELA ANA". Below it, several other participants are visible in smaller thumbnails, including "Nicușor Voinea", "Cristian Nanta", "Bianca", "Preda Mircea", "NICOLETA MARINSCU", "MATTEO GALIANI", and "You". A "20 others" icon is also present.

At the bottom of the Zoom window, the title bar reads "1:02 PM | MID-DAY: Matematica Întâlnește Industria". The Windows taskbar is visible at the very bottom, showing the time as 1:02 PM and the date as 13/02/2021.

1.8: Évaluation du MID-Day: Commentaires d'élèves

Après avoir terminé les MID-Days dans les cinq pays mentionnés ci-dessus, les participants à l'événement ont reçu un questionnaire pour évaluer leur satisfaction sur leur MID-Day.

Dans l'ensemble, les commentaires des étudiants ont été très positifs et encourageants. Les élèves ont exprimé leur satisfaction d'avoir l'opportunité de travailler en équipe et de se voir attribuer des rôles au sein de leurs équipes. Ils trouvaient cela agréable et constructif. Les élèves ont déclaré avoir le plus bénéficié du MID-Day pour diverses raisons. L'une des raisons était qu'ils ont travaillé sur un problème de la vie réelle en tant que membre d'une équipe, qu'ils ont développé leurs compétences en communication et en présentation, qu'ils ont réalisé à quel point les problèmes de la vie réelle sont compliqués et comment les mathématiques peuvent parfois leur apporter une solution. Les élèves ont également mentionné que la possibilité de présenter leur travail au format MID Day s'est avérée importante pour eux - ils ont pu ainsi améliorer et développer leurs compétences sociales telles que : l'échange d'informations, l'argumentation. Enfin, tous les élèves ont mentionné que c'était une expérience unique pour eux de rencontrer le monde de l'industrie !

Ce qu'il est important de mentionner, c'est que tous les élèves de tous les pays ont convenu que les problèmes du MID-Day étaient une manière d'apprendre beaucoup plus intéressante et interactive que les problèmes mathématiques auxquels ils sont confrontés dans leurs écoles. De plus, la majorité des élèves ont mentionné qu'ils avaient réussi à apprendre quelque chose de nouveau grâce au MID-Day.

Lorsque les élèves ont été interrogés sur les avantages, la praticité et les difficultés de l'adoption de cela dans les programmes scolaires, certains ont déclaré qu'il était nécessaire d'avoir ce genre de problèmes dans les programmes scolaires et que les étudiants gagneraient en confiance dans les mathématiques en travaillant en équipe et acquerraient une véritable expérience professionnelle. Ils croient qu'il ne serait pas spécialement difficile d'adopter cette culture à l'école. D'autre part, certains élèves ont exprimé l'opinion que même si pour eux c'était une expérience formidable, que le niveau scolaire en mathématiques est généralement faible et que cette très belle expérience, dans les écoles où l'intérêt pour les mathématiques est faible pourraient au contraire aliéner davantage certains élèves.

En ce qui concerne certaines idées visant à améliorer le MID-Day, les élèves aimeraient voir plus d'options de problèmes à l'avenir avec des difficultés variables. Ils ont trouvé les problèmes et l'ensemble du processus très difficiles, et ils ont exprimé leur souhait que ces types d'événements et d'activités soient inclus dans les programmes scolaires dans la mesure du possible.

Partea C: Matematica întâlnește industria în Modelul Școlar – Îndrumări pentru implementarea Zilei Industriei Matematicii (MID-Day)

Capitolul 1: Îndrumări pentru MID-Day

Mai întâi trebuie să identificați elevii talentați și apoi să convingeți o industrie/companie de faptul că abilitățile lor de gândire laterală ar putea merita încercate. În același timp, trebuie să motivați cadrele universitare și profesorii să vă susțină și apoi trebuie să planificați și să proiectați un Eveniment Matematica întâlnește Industria (MID-Day). Acest capitol vă va oferi îndrumările necesare pentru planificarea și desfășurarea unui eveniment de tipul MID-Day.

1.1: Definiție și Glosar

Partener: În aceste Îndrumări, când discutăm despre parteneri, ne referim la o industrie pregătită să investească o zi sau două discutând cu cadre universitare, academicieni, profesori și elevi pentru a defini și pentru a aborda o problemă industrială.

Problemă industrială: O problemă adecvată prezentată de partenerul industrial, care este ușor de înțeles de către elevi, profesori și de către cadrele universitare. Această problemă trebuie să fie adaptată de ambele părți, trebuie să abordeze unele probleme utile ale industriei respective și să fie o problemă care prezintă provocări, dar care să fie fezabilă pentru elevi. Problema aleasă este rezultatul unui brainstorming comun între partenerul industrial, cadrele universitare și profesori, înainte de organizarea MID-Day. O listă a problemelor posibile este furnizată în secțiunea 1.4 de mai jos.

MID-Day: O adevărată zi a matematicii și a industriei, când elevi abordează o problemă stabilită de un partener industrial și adaptată de cadrele universitare și de către profesori. Ar putea fi mai mult o idee, rafinamentul unei idei, mai degrabă decât de fapt soluția completă a unei probleme.

1.2: Metodologie, cerințe și îndrumări pentru profesori (profesorul în calitate de mentor și co-creator)

Responsabilitățile profesorilor sunt selectarea elevilor și pregătirea lor într-un mod care să-i facă apti pentru un eveniment MID-Day fructuos. În cadrul Evenimentului MID-Day, profesorii trebuie să faciliteze munca elevilor.

* (MID-Day) - Evenimentul Matematica întâlnește Industria

Aceasta înseamnă că mai întâi trebuie să selectați elevii, fie baza notelor obținute la școală, la olimpiade sau prin selecția cercurilor de matematică... prin intermediul unui test scris, fie pe baza preselecției realizate de către asociația de părinți, pe De asemenea, trebuie să obțineți acorduri oficiale de la părinți și de la școli pentru participarea tuturor.

După prima discuție cu partenerul industrial și cu cadrele universitare, trebuie să vă decideți asupra tipului de MID-Day pe care doriți să o desfășurați. Veți găsi mai jos câteva propuneri. Va trebui să realizați un curs propriu pe baza unui plan de lecție pe care vi-l punem la dispoziție, pentru a aduce elevii la nivelul adecvat. Acest lucru implică, de asemenea, adaptarea rolurilor în funcție de niveluri: unii elevi mai avansați sau maturi pot acționa ca antrenori sau tutori pentru cei mai tineri, urmându-se astfel mai degrabă modelul *profesorului – ghid (modelul centrat pe elev)* decât modelul *profesorului expert (modelul tradițional, centrat pe profesor)*.

Facilitarea în timpul MID-Day implică susținerea elevilor timizi, susținerea încrederii în sine, reasigurarea acestora de faptul că cunoștințele lor nu sunt insuficiente, pentru ca ei să poată începe să se gândească la problemă și pentru ușurarea comunicării și asigurarea faptului că nimeni nu se simte nesigur, că nimeni nu-și impune punctul de vedere în mod unilateral, că fiecare voce se face auzită, că fiecare elev se simte util și apreciat într-un fel sau altul. Împărțirea problemei în sub-sarcini trebuie planificată din timp în acest scop.

Ca ultim pas ar trebui să raportați cum au decurs lucrurile, respectând regulamentul și auto-reglarea lor, modul în care a reacționat fiecare elev, ce anume declanșează sau împiedică motivația sa, cum s-a integrat, ce a fost propus și ce a fost creat.

După MID-Day, va trebui să acordați o atenție deosebită modului în care materialul creat în timpul MID-Day poate fi utilizat în continuare în cursul zilnic de matematică, relaționându-l cu cunoștințele instituționale. Căutați în continuare oportunități pentru ca elevii să-și împărtășească munca cu alții în târguri, competiții, forumuri, cluburi, sau să organizeze o expoziție în școala dumneavoastră. Acest lucru va ajuta la recrutarea elevilor pentru anul următor. Nu uitați să menționați că participarea la MID-Day poate fi utilizată de elevi în CV-ul lor atunci când solicită burse. Acest eveniment va crea și va dezvolta rețeaua lor profesională!

1.3: Metodologie, cerințe și îndrumări pentru universitățile care susțin evenimentele MID-Day

Partenerii din rândul universităților ar trebui să construiască echipa de studenți care vor lucra la problema industrială. Aceasta înseamnă că vor trebui să lucreze cu elevi, dar și la formarea elevilor privind gestionarea muncii în echipă. În colaborare cu partenerii industriali și cu profesorii, aceștia trebuie să adapteze problema într-un mod care aceasta să poată fi rezolvată de către elevi.

Responsabilitatea ta principală este să înființați, să organizați și să gestionați MID-Day și întâlnirile pregătitoare ce preced această zi. O interacțiune tipică cu profesorii și elevii va avea trei faze:

În **prima întâlnire**: Veți cunoaște într-un mod informal elevii și pe profesorii lor, îi veți motiva, veți construi o echipă în jurul materialului matematic original, creativ și distractiv. Îi puteți contacta pe cei care participă la clubul de matematică, la târgul de știință, la ziua matematicii, la competițiile de matematică, la întâlnirile inter-școlare și așa mai departe.

În **a doua întâlnire**, vor lucra la nivelul elevilor selectați pentru a construi grupul ca echipă. Asta înseamnă că îi vei învăța instrumentele pentru a lucra în echipă (vezi mai jos).

A treia întâlnire va consta în MID-Day. Va trebui să selectați locații convenabile și suficient de mari pentru întâlnire, fie la universitate sau la sediul partenerului industrial, să aveți la îndemână cele necesare pentru pauze de cafea și prânz și orice alte elemente necesare. O versiune hibridă, cu teleconferințe online, ar putea fi în regulă dacă nu este posibilă o întâlnire fizică posibilă.

Va trebui să facilitați comunicarea și munca în echipă, punând întrebările potrivite, astfel încât toată lumea să se simtă integrată și productivă. Aveți grijă să nu căutați singuri răspunsuri fără să vă bazați pe elevi. Asigurați-vă că elevii se fac relevanți și că se află în centrul MID-Day. Nu participați în calitate de profesor care predă elevilor, ei trebuie să fie văzuți drept colegi cercetători stagieri.¹⁰

Instrumente de bază pentru lucrul în echipă:

- Clarificați divergențele. Păstrând un mediu sigur, ajungeți la o dezbatere fructuoasă: nu votați, nu faceți compromisuri, nu alegeți la întâmplare, evitați medierea și negocierea, dar luptați pentru unanimitate.
- Ascultați și implicați-vă: nu lăsați pe nimeni să-și impună punctul de vedere, ci să argumenteze, nu respingeți, ci propuneți respingeri.
- Nu personalizați dezbaterile, nu luați lucrurile personal. Căutați soluții.
- Acceptați soluțiile numai dacă sunt înțelese și împărtășite de toată lumea.
- Oferirea unei instruirii ușoare în aceste teme, atât pentru elevi, cât și pentru profesori, crește probabilitatea de a obține soluții originale și de a constitui echipă productivă.

¹⁰ Hall, Jay (1971) Decizii, decizii, decizii, Psihologia Astăzi, Noiembrie, 51-54, 84-88.

1.4: Exemple propuse de evenimente MID-Day

- Discutați cu partenerul dumneavoastră industrial. Problemele de optimizare pot fi întâlnite în multe afaceri: fie că partenerul industrial este din domeniul tăierii sticlei, controlului numeric al roboților, gestionării unei case de depozitare sau amenajării unui lanț de aprovizionare, cu siguranță au la îndemână o problemă de optimizare deschisă, de bază, care trebuie clarificată dacă nu cumva chiar rezolvată.
- Un **Sondaj de Cercetare Statistică** îl va interesa pe partenerul dumneavoastră industrial, astfel acesta ar putea dori să cunoască mai bine nevoile clienților sau angajaților lor, iar un proiect de cercetare de piață sau de resurse umane este relativ ușor de configurat, distractiv și creativ de proiectat și poate duce la rezultate neașteptate, când este văzut dintr-un punct de vedere neconvențional: „*cum să transformăm acest spațiu deschis sumbru în contextul covid?*”. Un plan de lecție în statistică și metodologia de anchetă este în regulă înainte de a trece efectiv la treabă.
- Un **Traseu Științific** pe amplasamentul industrial este o modalitate excelentă pentru industrie de a-și promova munca în timpul vizitelor cu clasa și pentru elevi să dezvolte gândire critică pentru soluționarea problemelor și să privească împrejurimile lor cu un ochi științific: „*cât de mare este acest rezervor? cât timp va dura să-l umpleți?*” Planul de lecție ar trebui să se concentreze pe metodologia de modelare și măsurare, descoperind, de exemplu, că propriul corp este un instrument destul de precis pentru estimările cantităților de zi cu zi.
- Dacă nu aveți contact direct cu un partener industrial, nu uitați de **industria educațională** și cereți elevilor să creeze jocuri pedagogice pentru dumneavoastră: lăsați-i să inventeze noi moduri de a preda prin *gamificarea* unei părți din conținutul dumneavoastră. Învățați de la ei! Planul de lecție ar trebui să predea bazele neuronale ale cogniției, *gamification*, determinarea scopurilor și sub-scopurilor, definirea provocărilor și nivelurilor, modalități de colaborare, ajutând elevii să-și înțeleagă mai bine propriul proces cognitiv. Și niciodată nu înțelegi un concept mai bine decât prin a trebui să-l explici, mai ales unui computer!

Odată stabilită tema principală, puteți planifica traseul lecției pentru a-i aduce pe elevi de unde se află la competențele cerute în ceea ce privește sarcinile depuse. Adică: trebuie să conduci lecția într-o prezentare de proiect. Această primă zi de școală ar trebui să fie relaxată și plină de bucurie, elevii lucrând în echipă și cunoscându-se între ei.

Când problema este definită și adaptată cu partenerul, ar trebui să o trimiteți elevilor în prealabil cu posibilitatea de a reacționa, de a interacționa, cu întrebări clarificatoare.

În ziua cea mare MID-Day, elevii vizitează șantierul industrial/locația industrială sau cel puțin discută cu reprezentanții industriei și lucrează împreună la proiectul lor în cursul dimineții.

Iată un posibil program pentru MID-Day

- 9h-9:30: Bine ai venit, conversație pe teme obișnuite, spargerea gheții
- 9:30-10:30: Team building cu o activitate de grup precum „lost on the moon” sau o activitate similară. <https://netmind.net/play-the-moon-landing-exercise/>
- 10:30-10:45 Pauză
- 10:45-11:45 Discuție în subgrupuri, cu privire la problema zilei de mijloc, înțelegerea problemei, primele piste ale gândurilor...
- 11:45-12:15 Prima sinteză asupra strategiilor posibile de abordare a problemelor, stabilirea echipajelor pe sarcini specifice.
- 12:15-13:45 Pauza de prânz
- 13:45-15:45 Lucrați în subgrupuri pe strategii și sarcini specifice.
- 15:45-16 Pauză
- 16-16:30 Pregătirea sintezei și prezentarea acesteia [Elevii]
- 16:30-17:15 Prezentare în fața partenerilor industriali, discuție

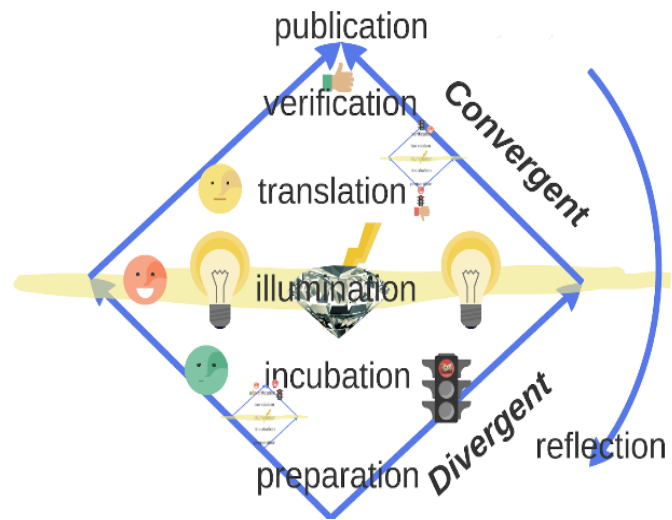
1.5: Rezultate de evaluare, Ce să faceți și ce să nu faceți

În timpul MID-Day:

Atenție că fiecare elev ar trebui să aparțină unui grup mic, de cel mult 8 elevi, care urmează să fie format în activitățile pregătitoare, de exemplu cu elevii mai mari care supraveghează grupul în calitate de antrenor, făcându-i pe toți să se simtă utili și integrați. O atenție deosebită trebuie acordată elevilor nesiguri punându-le întrebări adecvate, evidențiind și dezvoltând abilitățile speciale ale fiecărui elev, sporind încrederea lor în sine. Problema industrială este astfel analizată din fiecare perspectivă, astfel încât să fie posibilă împărțirea sarcinilor parțiale între toți membrii echipei, favorizând colaborarea tuturor membrilor echipei.

Evaluarea ar trebui să se concentreze pe aceste aspecte: faptul că fiecare elev s-a simțit luat în considerare ca gânditor și creator individual. Desigur, nu toate ideile pot duce la un produs de succes și există posibilitatea să nu rezulte niciun rezultat practic produs din MID-Day. Această situație nu trebuie resimțită ca un eșec, ci trebuie evidențiate potențialitățile, precum și creativitatea ideilor și producțiilor. Dacă aveți nevoie de numere, sau doriți să atribuiți trofee individuale, puteți aborda evaluarea creativității procesului de idee care a avut loc de-a lungul celor patru dimensiuni descrise de Guilford: Fluiditate, Flexibilitate, Elaborare și Originalitate:

- Fluiditatea este capacitatea de a produce mult, de a elabora multe exemple.
- Flexibilitatea este capacitatea de a schimba o strategie atunci când sunteți blocat, serendipitatea și adaptarea fiind în mod clar cuvintele cheie.
- Elaborarea cercetează în profunzime o întrebare, caută detaliile fine, este exhaustivă în căutarea ei.
- Originalitatea este tipul de creativitate pe care îl recunoașteți când îl vedeți: nu ați văzut niciodată așa ceva, este nou și provocator.



Întrucât vorbim de ideație, fiți conștienți că nu suntem în mintea elevilor. Pentru a fi pe deplin creativă, o idee trebuie să treacă prin tot ciclul creativității cu cele două momente de fază divergente și convergente: de la pregătire până la incubație, apoi iluminarea (momentul "Aha!"), dar această idee trebuie să iasă din mintea elevului și tradusă în cuvinte, schițe și așa mai departe, pentru a fi verificată, apoi publicată, propusă colegilor să vadă ce părere au despre ea și dacă o validează. Și în cele din urmă această idee poate fi reflectată și integrată într-un ciclu mai mare de idei pentru a aborda problema care este abordată.

Nu ar trebui să confundăm creativitatea doar cu gândirea divergentă: o fază de brainstorming este ok, dar la un moment dat, trebuie să se îndepărteze de la o simplă visare și să convergă către o soluție reală viabilă și este nevoie de cât mai multă creativitate în această fază convergentă. De asemenea, momentul de iluminare Aha nu trebuie supraevaluat, poezia și industria se pot amesteca doar până la un punct, ceea ce pare a fi o idee originală trebuie să se traducă și în scheme reale. Aceste puncte subtile trebuie luate în considerare de către echipă pentru a stimula o mentalitate productivă în rândul elevilor.

După MID-Day:

Nu se vor afișa imediat toate rezultatele în timpul MID-Day. elevii se vor întoarce acasă cu o mulțime de observații, idei și planuri. În ultimă fază, această problemă poate fi reluată și prezentată din nou partenerului industrial. Este important să se organizeze o modalitate prin care ei să facă schimb cu ceilalți. Acesta poate fi folosit în continuare, pentru târguri școlare, forumuri, înfrățire a școlilor, săptămâni de integrare, competiții și așa mai departe, pentru a se prezenta ceea ce s-a realizat.

1.6: De ce ar trebui să se implice industria, beneficii pentru industrie

Acest punct de interes al partenerului industrial trebuie clarificat în prealabil, dar este bine să îl revedeți la sfârșitul zilei de mijloc și să parcurgeți lista de verificare: Au beneficiat toate părțile de pe urma MID-Day? Gândirea laterală a deblocat cu adevărat o situație de creativitate sau pur și simplu a reușit industria să-și arate afacerile către persoane din afară? Beneficiile sunt într-adevăr diferite în funcție de tipul de MID-Day organizată. Pentru a parcurge exemplele generice, am dat:

- Sondajul de cercetare statistică a reușit să evidențieze elemente nevăzute înainte sau a fost pur și simplu o modalitate de a comunica, fie intern, fie cu potențialii clienți? A permite clienților sau angajaților mai buni să-și exprime opiniile și să vadă că acestea sunt luate în considerare reprezintă deja un aspect ce poate fi pus în valoare.
- Este traseul științific distractiv și informativ? Poate fi folosit cu potențiali clienți, sau în cursuri? Le deschide ochii participanților asupra provocărilor pe care le abordează industria? În cazul unei vizite turistice a unei locații, partenerul descoperă locația cu ochi noi? Va servi scopului de turism științific?
- În ceea ce privește industria educațională, configurația elevilor dispune de jocuri pedagogice interesante, informative și distractive? Au învățat ceva elevii din proiectarea jocului pedagogic?
- Industria va avea oportunitatea de a intra în contact cu elevi talentați și, eventual, de a lucra cu aceștia în viitor.

1.7: Implementarea MID-Day – Evenimente MID-Day în diverse țări:

Evenimentul MID-Day a fost realizat în variantă pilot și a fost implementat în cinci țări europene; Acestea sunt: Cipru, Franța, Germania, Polonia și România. Ideea din spatele evenimentului MID-Day este că aplicarea matematicii în viața reală este o necesitate absolută pentru orice curriculum din zilele noastre și chiar mai mult pentru elevii supradotați care pot contribui la inovarea care ar putea face viața umană mai bună și, în același timp, promovează antreprenoriatul în programa școlară. Evenimentul Matematica întâlnește l'industria este conceput pentru a oferi elevilor probleme provocatoare nerezolvate de industrie din viața reală, cu scopul de a lucra fie în echipă, fie individual și cu scopul de a folosi matematica pentru a propune un set de soluții posibile la problemele date. În plus, elevii au avut ocazia să colaboreze unii cu alții și să lucreze împreună pentru un obiectiv comun, să cunoască cercetători din învățământul superior, precum și oameni din industrie și profesori de liceu să vadă, să obțină idei și să se inspire cu privire la modul în care astfel de probleme ar putea fi utilizate ca parte a predării lor obișnuite.

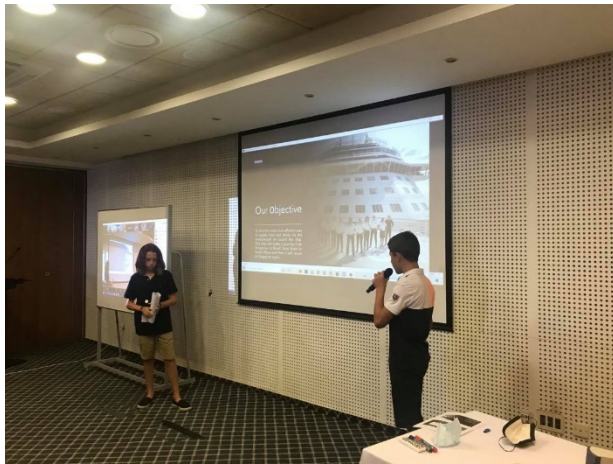
Pregătirea și implementarea evenimentului au variat în fiecare țară. Au fost prezentate diverse probleme din industrie, pe care elevii au fost provocați să le rezolve. Prin urmare, mai jos vom analiza pregătirea și implementarea fiecărui eveniment al MID-Day pe fiecare țară. În acest fel, pot fi extrase exemple și sugestii privind metodologia MID-Day.

Situația organizării evenimentului în Cipru

Acțiuni de pregătire: Pregătirea pentru acest eveniment a început cu două luni înainte, când a fost trimisă o scrisoare de invitație/motivare către elevii cărora le place matematica și care ar fi putut fi interesați de o astfel de activitate. Între timp, echipa de profesori INNOMATH a Școlii private Heritage a contactat companiile locale, în urma organizării unui Focus Group, și le-a cerut să se alăture acestui eveniment punând la dispoziție elevilor o problemă reală din industrie. Două companii au furnizat fiecare câte o problemă în scopul MID-Day. Problema A a fost furnizată de V-ships, care este una dintre cele mai mari companii de transport maritim din lume. Problema descria un caz specific al unei nave-cisternă care face comerț între Singapore și Brazilia, care trebuie să fie aprovizionată cu provizii pentru a avea provizii la bord în orice moment. În acest proiect, elevii trebuiau să acționeze în beneficiul furnizorului de catering în încercarea de a formula cel mai rentabil plan de reaprovizionare, asigurându-se că tarifele convenite sunt îndeplinite și că nava va avea stoc suficient la bord în orice moment în timpul călătoriei și în același timp trebuiau să maximizeze profitul pentru compania furnizorului de catering. Problema B a fost prezentată de o firmă de arhitectură, iar elevilor li s-a cerut să ia locul unui arhitect, să înțeleagă procesul și să găsească cea mai bună soluție posibilă pentru a servi un client în funcție de cerințele casei sale, de materialele ce trebuie utilizate și de bugetul disponibil.

Pregătirea de către elevi a prezentărilor pentru MID-Day: Mulți elevi care iubesc matematica, au optat pentru a participa la activitatea MID-Day. A fost o oportunitate pentru ei de a gândi creativ și neconvențional, fie individual, fie în echipe, pentru a găsi posibile soluții la problemele date într-o activitate în afara clasei. Din cauza restricțiilor Covid19 și a elevilor participanți veniți din diferite orașe, au fost organizate două întâlniri online pentru profesorii Heritage, elevii participanți, reprezentanții companiei și Societatea de Matematică. Întâlnirile online au avut loc la începutul și la mijlocul lunii iulie. Scopul întâlnirilor a fost ca elevii să-și creeze echipele, să se familiarizeze cu problemele, să obțină explicații și lămuriri suplimentare de la profesori, precum și de la reprezentanții companiei. Au fost, de asemenea, prezentate linii directe generale pentru buna desfășurare a MID-Day și cerințele pentru prezentările elevilor. După prima întâlnire, elevii au avut câteva zile pentru a decide problema preferată și au început să lucreze la ea. Problema A a fost mai populară deoarece a fost selectată de toți elevii. Apoi, a avut loc o altă întâlnire online pentru elevi și profesori pentru a discuta progresul în proiectarea soluției. Elevii au fost sfătuiți despre cum să-și organizeze munca și cum să împartă volumul de lucru în cadrul echipei. De asemenea, au avut șansa de a cere mai multe sfaturi și clarificări care i-au putut ajuta să rezolve soluția propusă.

Ziua efectivă MID-Day: Miercuri, 28 iulie 2021, evenimentul a avut loc la Agros în prezența elevilor, părinților, profesorilor, cercetătorilor din învățământul superior și partenerilor de proiect. Elevii și-au prezentat posibilele soluții, iar la finalul fiecărei prezentări reprezentantul V-ships, domnul Loucas Chrysochos, și-a oferit feedback-ul cu privire la soluțiile propuse de elevi. De asemenea, toți ceilalți participanți au fost liberi să pună întrebări elevilor. Toate soluțiile primite de la elevi au fost diferite și acest lucru a inițiat o discuție fructuoasă între elevii participanți și public. La final, reprezentantul industriei și-a exprimat entuziasmul pentru munca asiduă a elevilor, abilitățile excelente de prezentare și modul analitic de a-și prezenta soluția. **El a menționat că a fost o experiență grozavă pentru el să participe la eveniment și să cunoască astfel de elevi care au făcut o treabă grozavă. În plus, și-a exprimat interesul de a lucra cu ei în viitor.**



Situația organizării evenimentului în Franța

Ziua efectivă MID-Day:

Elevii care participă la MID-Day poartă două pălării, sunt conștienți că nu doar participă, ci și testează planul pedagogic inovator. Elevii au fost încadrați într-o mentalitate creativă, în care matematica era privită ca un instrument concret de rezolvare a problemelor din viața reală cu valoare economică.

Elevii au fost lăsați să se formeze în echipe prin elemente pentru spargerea gheții, apoi au instalat aplicația MathCityMap și au rulat traseul local de matematică.

Au petrecut cele mai bune zile pe teren, în cartierul renaștant, aflat în patrimoniul UNESCO, ascultând poveștile pe care le spunea ghidul profesionist, luând notițe, punând lucrurile în echipă, încercând idei, lăsând alți membri ai echipei să verifice fezabilitatea și natura distractivă a sarcinilor... Înapoi la universitate, sarcinile au fost codificate în sistemul MathCityMap.

Problema principală a fost să răspundă nevoilor reprezentantului industriei turismului: elevii au discutat despre cele mai bune modalități de a aborda diferite ținte, familii, săli de clasă, grupuri de prieteni. Înțelegerea constrângerilor ghidului profesionist, infuzarea suficientă a distracției și a informațiilor revelatoare la întrebări științifice a fost scopul principal al traseului matematic urmat.

A doua parte a MID-Day a fost feedback-ul către partenerul industrial. Elevii au avut ocazia să facă echipă pentru a-și organiza discursul. Aceasta a avut loc în timpul întâlnirii proiectului transnațional care a avut loc în Lyon la mijlocul lui decembrie 2021 în campusul UCBL. În ziua de după traseu, elevii și-au justificat alegerile pe care le-au făcut în fața juriului compus din reprezentantul industriei și parteneri internaționali ai proiectului InnoMath.



Situația organizării evenimentului în Germania

Ațiuni de pregătire: Pregătirea pentru evenimentul MID-Day a început cu 6 luni înainte printr-o întâlnire cu partenerii din industrie care discutau sarcini potențiale. La întâlnirea Focus Group, care a constat din profesori, oameni de știință și parteneri din industrie s-au reunit și s-au evaluat ideile și programul propus pentru MID-Day. Ulterior, școlile cu focus pe matematică și științele naturii au fost invitate să participe la eveniment. Împreună cu Planetariul Stiftung Berlin, s-a decis să se pună elevilor problema modului în care schimbările anotimpurilor - schimbările climatice - pot fi făcute vizibile statistic și cum pot fi făcute predicții. În mod special, modelele utilizate urmau să fie evaluate și urma a se reflecta asupra acestora.

Pregătirea de către elevi a prezentărilor pentru MID-Day: 14 clase (420 de elevi) din școli cu accent pe matematică și științele naturii au participat la activitatea MID-Day. Din cauza restricțiilor Covid19, elevii au fost nevoiți să se întâlnească în școlile lor și acolo unde nu au putut să se întâlnească cu elevi din alte școli pentru pregătirea prezentării. Pregătirile au avut loc în luna septembrie, iar elevilor li s-a oferit material pentru a se familiariza cu problemele, material care a fost pregătit de cercetători și de partenerii din industrie. Profesorii i-au susținut pe elevi și i-au supravegheat. Au fost date, de asemenea, îndrumările generale pentru buna desfășurare a evenimentului MID-Day și cerințele pentru prezentările elevilor. După prima întâlnire, elevii au avut câteva zile pentru a decide asupra soluției și prezentării lor, care a fost apoi discutată în

clasă. Elevii au decis cum să-și organizeze munca și cum să împartă volumul de lucru în cadrul echipelor. De asemenea, elevii au avut șansa de a cere mai multe sfaturi și clarificări care i-au putut ajuta să rezolve soluția propusă.



Situația organizării evenimentului în Polonia

Acțiuni de pregătire: Pregătirile au început cu o lună înainte. Elevii au trebuit să se familiarizeze cu sarcina prezentată de biroul de turism. Acesta a constat în construirea unui algoritm care să permită crearea unui plan de călătorie pentru elevii care vizitează Cracovia. Acest plan trebuia să țină cont de o serie de condiții, precum: bugetul, durata, vârsta participanților, alegerea atracțiilor opționale și altele.

Pregătirea de către elevi a prezentărilor pentru MID-Day: Întrucât la eveniment au participat grupuri de elevi din aceleași clase, pregătirile au avut loc în varianta staționară. Munca lor a constat în colectarea de date relevante, precum: prețurile biletelor, distanța dintre locații, tarifele de închiriere a autocarelor, prețurile transportului public și altele. În a doua fază a pregătirilor, sarcina lor a fost să creeze un program care să simuleze costurile evenimentului. Pentru pregătire au fost folosite atât metode tradiționale, cât și tehnologii informaționale moderne. O parte din lucrări au fost făcute și de la distanță. În faza finală, premergătoare evenimentului, membrii grupului s-au concentrat pe prezentarea realizărilor lor, și anume pe pregătirea și exersarea discursurilor.

Ziua efectivă MID-Day: Evenimentul a fost împărțit în trei părți: În prima parte, a fost prezentat proiectul INNOMATH, împreună cu obiectivele, ipotezele și efectele sale dezvoltate pe durata sa. A doua parte a constat în prezentarea lucrărilor elevilor. Toate grupurile au prezentat rezultatele pregătirilor lor. După fiecare dintre ele, experții și invitații au avut ocazia să-și exprime opiniile cu privire la prezentările pregătite. A treia parte a fost o discuție pe teme legate de proiect și impactul acestuia asupra abordării moderne a predării.



Situația organizării evenimentului în România:

Notă: Acest exemplu de MID-Day este scris din perspectiva elevilor

Pe parcursul proiectului INNOMATH am ajuns să experimentăm multe lucruri noi care ne-au arătat cum este viața dincolo de liceu și cum poate fi folosită matematica în situații reale. Pentru a înțelege mai bine cât de esențială este matematica aplicată pentru buna funcționare a întreprinderilor și a instalațiilor industriale, am pornit într-o călătorie pentru a încerca să rezolvăm o problemă cu care se confruntă o companie locală, utilizând cunoștințele noastre.

Astfel, am vizitat societatea Fluorocarbon Polymers și am vorbit cu directorul acesteia. El ne-a explicat că se produc și se expediază diverse piese și materiale care servesc diverse scopuri în industrie. Am continuat apoi să urmărim linia de producție de la început până la sfârșit, sperând să descoperim modalități în care compania pierde bani și/sau resurse, astfel încât să putem găsi o soluție. Sa dovedit destul de dificil să găsim probleme în procesul lor, dar în final, am decis să încercăm să optimizăm transportul materiilor prime pe care le folosește societatea și al produselor finite către cumpărătorii lor.

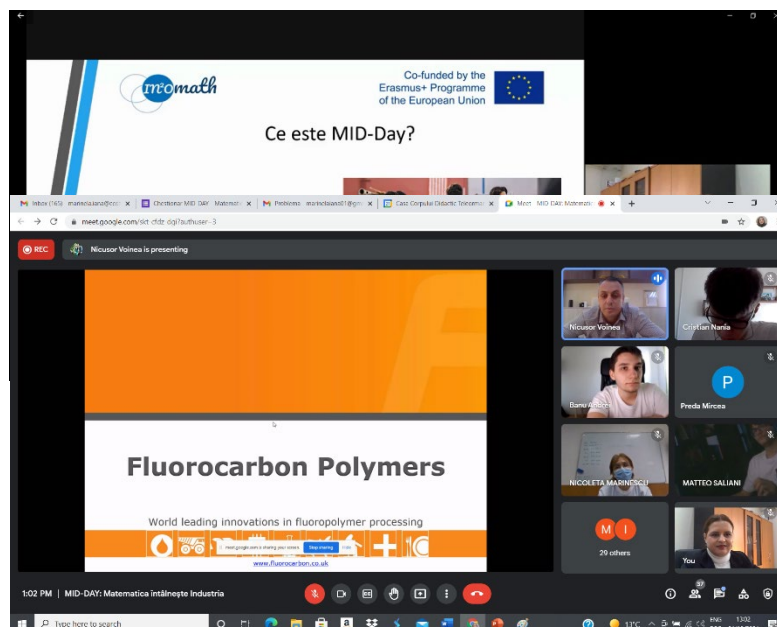
Problemele de optimizare a transporturilor sunt greu de rezolvat și necesită abilități de nivel universitar, prin urmare ceea ce am ajuns să facem a fost să învățăm niște algebră avansată care doar ne-a adus mai aproape de a putea rezolva astfel de probleme, ajutându-ne totodată să rezolvăm mai ușor problemele de algebră liniară de la nivel de liceu.

În clasă, profesorul nostru ne-a împărțit în două grupe: un grup care să învețe și să folosească metodele predate de obicei în liceu pentru calcularea matricilor inverse, determinarea rangului unei matrici și rezolvarea sistemelor de ecuații liniare; și un alt grup care să învețe să folosească metodele avansate. Scopul a fost de a compara diferitele metode la sfârșitul cursului și de a determina care sunt mai rapide și mai ușor de utilizat.

Pe măsură ce am exersat rezolvarea exercițiilor, am fost încurajați să ne ajutăm reciproc să înțelegem și să învățăm mai repede. Într-adevăr, uneori mi s-a părut mai ușor să pun întrebări colegilor mei despre noile concepte, mai degrabă decât să pun întrebări profesorului, iar perspectiva dată din punctul de vedere al unui elev a permis o abordare utilă asupra problemei. Cel mai plăcut aspect al acestei metode de învățare a fost faptul că elevii simt mai puțin că învață și mai mult că se joacă, pe măsură ce-și transmit idei unul celuilalt.

În cele din urmă, ambele echipe au prezentat și au schimbat metodele pe care le-au folosit pentru a înțelege cealaltă jumătate a acestui demers și pentru a putea trage concluzii mai ușor. Am fost cu toții de acord că metodele avansate nu au fost atât de greu de înțeles pe cât ne-am gândit anterior, dar contează foarte mult cum sunt învățate. Privind în urmă la tot ce am văzut și am realizat, mă simt bine că am ales să studiez ingineria la universitate și sunt mai bine pregătit să înfrunt lumea reală.

Ziua efectivă MID-Day:



1.8: Evaluarea evenimentelor MID-Day: Comentarii ale elevilor

După finalizarea evenimentelor MID-Day în cele cinci țări menționate mai sus, participanților la eveniment li s-a trimis o evaluare pentru a nota satisfacția lor față de activitățile MID-Day.

Feedback-ul general din partea elevilor a fost foarte pozitiv și încurajator. Elevii și-au exprimat satisfacția că au avut oportunitatea de a lucra ca membri într-o echipă și că li s-au atribuit roluri în cadrul echipelor lor. Au considerat că este plăcut și constructiv. Toți elevii din toate evenimentele au declarat că au beneficiat de activitatea MID-Day din diverse motive. Unul dintre motive a fost că au lucrat într-o problemă din viața reală ca membru al unei echipe, și-au dezvoltat abilitățile de comunicare și prezentare, și-au dat seama cât de complicate sunt problemele din viața reală și cum matematica le poate oferi o soluție. Elevii au mai menționat că oportunitatea de a-și prezenta lucrările în formatul MID-Day s-a dovedit a fi importantă pentru ei – și că astfel și-au putut îmbunătăți și dezvolta abilitățile sociale precum: schimb de informații, argumentare. În cele din urmă, toți elevii au menționat că a fost o experiență unică pentru a cunoaște lumea industriei!

Ceea ce este important de menționat este că toți elevii din toate țările au fost de acord că problemele abordate în evenimentele MID-Day sunt o modalitate mult mai interesantă și mai interactivă de a învăța decât problemele matematice cu care se confruntă în școlile lor. În plus, majoritatea elevilor au menționat că au reușit să învețe ceva nou din evenimentul MID-Day.

Când elevii au fost întrebați despre părerea lor cu privire la avantajele, caracterul practic și dificultățile adoptării acestui mod în programa școlară, unii au spus că este necesar să existe acest tip de probleme în programa școlară și că elevii vor câștiga mai multă încredere lucrând la matematică în echipă și dobândind experiență de lucru reală. Ei cred că nu ar fi greu să se adopte această cultură în școală. Pe de altă parte, unii elevi și-au exprimat părerea că, deși pentru ei aceasta a fost o experiență grozavă, că nivelul școlii de matematică este scăzut și, în ciuda experienței foarte plăcute, în școli interesul pentru matematică este scăzut, iar sarcini mai mari de acest gen i-ar putea îndepărta și mai mult pe unii elevi de matematică.

În ceea ce privește unele idei pentru îmbunătățirea MID-Day, elevii ar dori să vadă mai multe opțiuni de probleme în viitor, cu diferite niveluri de dificultate. Ei au considerat că problemele și întregul proces sunt foarte provocatoare și și-au exprimat dorința ca astfel de evenimente și activități să fie incluse în programa școlară acolo unde este posibil.

Part D: Communication channels

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Useful Links: <http://kleverkids.de/> , <https://bund-hochbegabung.de/>

